



A new approach for the study of limit cycles

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Abstract

We prove that star-like limit cycles of any planar polynomial system can also be seen either as solutions defined on a given interval of a new associated planar non-autonomous polynomial system or as heteroclinic solutions of a 3-dimensional polynomial system. We illustrate these points of view with several examples. One of the key ideas in our approach is to decompose the periodic solutions as the sum of two suitable functions. As a first application we use these new approaches to prove that all star-like reversible limit cycles are algebraic. As a second application we introduce a function whose zeroes control the periodic orbits that persist as limit cycles when we perturb a star-like reversible center. As far as we know this is the first time that this question is solved in full generality. Somehow, this function plays a similar role than an Abelian integral for studying perturbations of Hamiltonian systems.

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