Physics of the Dark Universe 38 (2022) 101139

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Global dynamics of the Hořava–Lifshitz cosmology in the presence of non-zero cosmological constant in a flat space

Fabao Gao [a,b,*], Jaume Llibre [b]

[a] School of Mathematical Science, Yangzhou University, Yangzhou 225002, China
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra 08193, Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:
Received 22 October 2021
Received in revised form 10 November 2022
Accepted 11 November 2022

Keywords:
Global dynamics
Hořava–Lifshitz
Non-zero cosmological constant

ABSTRACT

Using the qualitative theory of differential equations, the global dynamics of a cosmological model based on Hořava–Lifshitz gravity is studied in the space with zero curvature in the presence of the non-zero cosmological constant. The study shows that there may be three unstable finite equilibrium positions under the Hořava–Lifshitz cosmology model and that the final evolution of the orbits of the cosmological model in the physical region of interest may tend towards some infinite equilibrium position, which may correspond to the late-time state of the universe.

© 2022 Elsevier B.V. All rights reserved.

1. Introduction

A decade ago Hořava [1] brought forward a new theory on space–time asymmetric gravitation, called Hořava–Lifshitz gravity, together with the scalar field theory of Lifshitz. This theory's applications to cosmology, dark energy, and black holes have stimulated many studies (See review papers [2,3] or regular literature [4–21]).

Based on whether the value of Λ (cosmological constant) is zero and the flatness of the universe, i.e. whether the space curvature k is equal to zero, Leon et al. [10–12,18] divided Hořava–Lifshitz cosmology into four cases: (1) $\Lambda = 0, k = 0$; (2) $\Lambda = 0, k \neq 0$; (3) $\Lambda \neq 0, k = 0$; (4) $\Lambda \neq 0, k \neq 0$ under the classic FLRW metric. They either studied partially the three-dimensional dynamics of Hořava–Lifshitz cosmology, or analyzed its two-dimensional dynamics under exponential potentials.

For the cosmological constant Λ that scientists have been concerning about, Carlip [22] argued that the vacuum fluctuations under the standard effective field theory produce a huge Λ and produce high k on the Planck scale, but it is almost invisible at the observable scale. Compared with the prediction in the standard Λ cold dark matter model, Valentino et al. [23] proposed that the cosmological space may be a closed three-dimensional sphere, i.e., the cosmological space's curvature may be positive, based on the enhanced lensing amplitude in the cosmic microwave background power spectra confirmed by Planck Legacy 2018 release [24]. Although this study provides the latest results, the debate about the universe's shape has not yet been settled. An important reason is that the calculation of the critical density of the universe depends on the measurement of the Hubble constant, which is different as estimated from different cosmological data. Then the boundary line of the universe is fuzzy. So it is too early to say that the universe must be closed.

The global dynamics of the Hořava–Lifshitz cosmology under the background of FLRW with $k = 0, \Lambda = 0$ was studied in [7], and the case of $k \neq 0, \Lambda = 0$ has also been addressed in [8]. In this paper we will consider the flat universe with $\Lambda \neq 0$. More precisely, in this paper we present the global dynamics of the Hořava–Lifshitz cosmology in the three-dimensional space including the infinity, neither partial or local dynamics, nor the two-dimensional case. The motivation for considering the cosmological constant and scalar field here is to fully explain the universe's dynamic evolution and final state under the Hořava–Lifshitz gravitational model.

2. The cosmological equations

In order to describe the cosmological model, we first briefly review the Hořava–Lifshitz theory of gravity proposed in [1]. The field content in this theory can be derived from the space vector $\mathfrak{N}_i$ and scalar $\mathfrak{N}$, see [12,13]. They are actually common 'lapse' and 'shift' variables in general relativity. From this the complete metric can be expressed as

$$ds^2 = -\mathfrak{N}^2 dt^2 + g_{ij}(dx^i + \mathfrak{N}^i dt)(dx^j + \mathfrak{N}^j dt), \quad \mathfrak{N}_i = g_{ij}\mathfrak{N}^j, \qquad (1)$$

where g_{ij} is a spatial metric, here i and j are natural numbers from 1 to 3. The coordinate transformations follow $t \to l^3 t,\ x^i \to l x^i$. Note that g_{ij} is invariant, the same as $\mathfrak{N}$, but $\mathfrak{N}^i$ is scaled to $l^{-2}\mathfrak{N}_i$.

* Corresponding author at: School of Mathematical Science, Yangzhou University, Yangzhou 225002, China.
E-mail addresses: gaofabao@sina.com (F. Gao), jllibre@mat.uab.cat (J. Llibre).

https://doi.org/10.1016/j.dark.2022.101139
2212-6864/© 2022 Elsevier B.V. All rights reserved.