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Limit Cycles in Planar Piecewise Linear Hamiltonian Systems with Three Zones Without Equilibrium Points

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We study the existence of limit cycles in planar piecewise linear Hamiltonian systems with three zones without equilibrium points. In this scenario, we have shown that such systems have at most one crossing limit cycle.

Keywords: Piecewise smooth vector field; limit cycle; Hamiltonian vector field.

1. Introduction and Statement of the Main Results

The study of piecewise smooth vector fields, especially the planar case, has grown over the last 30 years mainly due to its great applicability as a mathematical model of a series of applied phenomena in relay systems, mechanics, electrical circuits, among others. As a landmark for such studies, we cited the book by Andronov *et al.* [1966], and more recently Filippov's book [Filippov, 1988] and the book by di Bernardo *et al.* [2008].

Many of the studies developed so far consider piecewise smooth vector fields with two zones and few studies have been done with more zones. A *piecewise smooth vector field with three zones* in the plane is composed of three C^r vector fields X, Y and Z, $r \geq 1$, defined on $\mathbb{R}^2$ and separated by a pair of disjoint connected unbounded smooth curves Σ_1 and Σ_2. The *separation curves* Σ_i are obtained by considering $\Sigma_i = h_i^{-1}(0)$, where $h_i : \mathbb{R}^2 \to \mathbb{R}$ are differentiable functions having 0 as a regular value, for $i = 1, 2$. Thus $\mathbb{R}^2 = \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3 \cup \Sigma_1 \cup \Sigma_2$, where the *zones* $\mathcal{R}_i$, $i = 1, 2, 3$, are unbounded disjoint regions defined on the complement of the separation curves. So, a piecewise smooth vector field with three zones in the plane can be written