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Global phase portraits of a predator–prey system

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Abstract. We classify the global dynamics of a family of Kolmogorov systems depending on three parameters which has ecological meaning as it modelizes a predator–prey system. We obtain all their topologically distinct global phase portraits in the positive quadrant of the Poincaré disc, so we provide all the possible distinct dynamics of these systems.

Keywords: predator–prey system, Kolmogorov system, global phase portrait, Poincaré disc.

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1 Introduction

Rosenzweig and MacArthur introduced in [25] the following predator–prey model

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - y\frac{mx}{b+x},$$
$$\dot{y} = y\left(-\delta + c\frac{mx}{b+x}\right),$$

where the dot as usual denotes derivative with respect to the time t, $x \geq 0$ denotes the prey density (#/unit of area) and $y \geq 0$ denotes the predator density (#/unit of area), the parameter $\delta > 0$ is the death rate of the predator, the function $mx/(b+x)$ is the # prey caught per predator per unit time, the function $x \to rx(1 - x/K)$ is the growth of the prey in the absence of predator, and $c > 0$ is the rate of conversion of prey to predator.

The Rosenzweig and MacArthur system is a particular system of the general predator–prey systems with a Holling type II, see [12, 13].

In [14] Huzak reduced the study of the Rosenzweig and MacArthur system to study a polynomial differential system. In order to do that the first step is to do the rescaling $(\overline{x}, \overline{y}, \overline{b}, \overline{c}, \overline{\delta}) = (x/K, (m/rK)y, b/K, cm/r, \delta/r)$. After denoting again $(\overline{x}, \overline{y}, \overline{b}, \overline{c}, \overline{\delta})$ by (x, y, b, c, δ) and doing

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