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Bifurcation of limit cycles in piecewise quadratic differential systems with an invariant straight line

Leonardo P.C. da Cruz [a], Joan Torregrosa [b,c,*]

[a] *Departamento de Matemática, Universidade Federal de São Carlos, Rodovia Washington Luís, Km 235, Caixa Postal 676, SI-2000, São Carlos (SP), Brazil*
[b] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*
[c] *Centre de Recerca Matemàtica, Campus de Bellaterra, 08193 Bellaterra, Barcelona, Spain*

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ABSTRACT

We solve the center-focus problem in a class of piecewise quadratic polynomial differential systems with an invariant straight line. The separation curve is also a straight line which is not invariant. We provide families having at the origin a weak-foci of maximal order. In the continuous class, the cyclicity problem is also solved, being 3 such maximal number. Moreover, for the discontinuous class but without sliding segment, we prove the existence of 7 limit cycles of small amplitude.

1. Introduction

In past years, a big interest in the study of the dynamics of piecewise systems has emerged, due to the fact that many real phenomena can be modeled with this class of systems. For example, the existence and uniqueness of periodic orbits or the existence of a continuum of periodic orbits. These problems appear in many areas of research. In particular in electrical and mechanical engineering, in control theory, and even in the analysis of genetic networks. See for example [1,10].

Usually, the simplest models are defined via planar piecewise polynomial vector fields $Z = (Z^+, Z^-)$ in the following way. Taking 0 as a regular value of the function $h : \mathbb{R}^2 \to \mathbb{R}$, we denote the discontinuity curve by $\Sigma = h^{-1}(0)$ and the two regions it delimits by $\Sigma^\pm = \{\pm h(x, y) > 0\}$. So, the piecewise vector field can be written as

$$Z^\pm : (\dot{x}, \dot{y}) = (X^\pm(x, y), Y^\pm(x, y)), \text{ for } (x, y) \in \Sigma^\pm, \tag{1}$$

* Corresponding author at: Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.
E-mail addresses: leonardo@mat.uab.cat (L.P.C. da Cruz), torre@mat.uab.cat (J. Torregrosa).

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