ISSN 1560-3547, Regular and Chaotic Dynamics, 2020, Vol. 25, No. 3, pp. 250–272. © Pleiades Publishing, Ltd., 2020.

On the Convex Central Configurations of the Symmetric $(\ell+2)$-body Problem

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Received January 22, 2020; revised April 20, 2020; accepted April 26, 2020

Abstract—For the 4-body problem there is the following conjecture: Given arbitrary positive masses, the planar 4-body problem has a unique convex central configuration for each ordering of the masses on its convex hull. Until now this conjecture has remained open. Our aim is to prove that this conjecture cannot be extended to the $(\ell+2)$-body problem with $\ell \geqslant 3$. In particular, we prove that the symmetric $(2n+1)$-body problem with masses $m_1 = \ldots = m_{2n-1} = 1$ and $m_{2n} = m_{2n+1} = m$ sufficiently small has at least two classes of convex central configuration when $n = 2$, five when $n = 3$, and four when $n = 4$. We conjecture that the $(2n+1)$-body problem has at least n classes of convex central configurations for $n > 4$ and we give some numerical evidence that the conjecture can be true. We also prove that the symmetric $(2n+2)$-body problem with masses $m_1 = \ldots = m_{2n} = 1$ and $m_{2n+1} = m_{2n+2} = m$ sufficiently small has at least three classes of convex central configuration when $n = 3$, two when $n = 4$, and three when $n = 5$. We also conjecture that the $(2n+2)$-body problem has at least $[(n+1)/2]$ classes of convex central configurations for $n > 5$ and we give some numerical evidences that the conjecture can be true.

MSC2010 numbers: 70F10, 70F15

DOI: 10.1134/S1560354720030028

Keywords: convex central configurations, $(\ell+2)$-body problem

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The equations for the classical Newtonian N-body problem are given by

$$m_i\,\ddot{\mathbf{q}}_i = \sum_{j=1, j\neq i}^{N} G\,m_i\,m_j\,\frac{\mathbf{q}_j - \mathbf{q}_i}{|\mathbf{q}_j - \mathbf{q}_i|^3},$$

$i = 1, \ldots, N$, where $\mathbf{q}_i = (x_i, y_i) \in \mathbb{R}^2$ is the position vector of the point mass m_i in an inertial coordinate system, $|\mathbf{q}_j - \mathbf{q}_i|$ is the Euclidean distance between the masses m_j and m_i, and G is the gravitational constant which can be taken to be equal to one by choosing conveniently the unit of time. The *configuration space* of the planar N-body problem is

$$\mathcal{E} = \{(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathbb{R}^{2N}\ :\ \mathbf{q}_i \neq \mathbf{q}_j,\ \text{for } i \neq j\}.$$

Given $m_1, \ldots, m_N$, a configuration $(\mathbf{q}_1, \ldots, \mathbf{q}_N) \in \mathcal{E}$ is *central* if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_i = -\lambda\,(\mathbf{q}_i - \mathbf{cm})$$

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