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Spatial Convex but Non-strictly Convex Double-Pyramidal Central Configurations of the $(2n + 2)$-Body Problem

Montserrat Corbera[1] · Jaume Llibre[2]

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Abstract

A configuration of the N bodies is convex if the convex hull of the positions of all the bodies in $\mathbb{R}^3$ does not contain in its interior any of these bodies. And a configuration is strictly convex if the convex hull of every subset of the N bodies is convex. Recently some authors have proved the existence of convex but non-strictly convex central configurations for some N-body problems. In this paper we prove the existence of a new family of spatial convex but non-strictly convex central configurations of the $(2n + 2)$-body problem.

Keywords Spatial central configuration · Convex but non-strictly convex central configurations · n-body problem

Mathematics Subject Classification 70F7 · 70F15

1 Introduction and Statement of the Main Result

The *equations of motion* of the spatial N-body problem are

$$m_k \, \ddot{\mathbf{q}}_k = - \sum_{\substack{j=1 \\ j \neq k}}^{N} G \, m_k \, m_j \, \frac{\mathbf{q}_k - \mathbf{q}_j}{|\mathbf{q}_k - \mathbf{q}_j|^3} \,,$$

for $k = 1, \ldots, N$, where G is the gravitational constant which will be taken equal to one by choosing conveniently the unit of time, $\mathbf{q}_k \in \mathbb{R}^3$ is the position vector of the punctual mass m_k in an inertial coordinate system, and the two dots denote the second derivative with respect to the time t.

✉ Montserrat Corbera
montserrat.corbera@uvic.cat

Jaume Llibre
jllibre@mat.uab.cat

1 Departament d'Enginyeries, Universitat de Vic–Universitat Central de Catalunya, C/ Laura 13, 08500 Vic, Barcelona, Spain

2 Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain