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Probability of Occurrence of Some Planar Random Quasi-homogeneous Vector Fields

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Abstract. The objective of this work is the study of the probability of occurrence of phase portraits in a family of planar quasi-homogeneous vector fields of quasi degree q, that is a natural extension of planar linear vector fields, which correspond to $q = 1$. We obtain the exact values of the corresponding probabilities in terms of a simple one-variable definite integral that only depends on q. This integral is explicitly computable in the linear case, recovering known results, and it can be expressed in terms of either complete elliptic integrals or of generalized hypergeometric functions in the non-linear one. Moreover, it appears a remarkable phenomenon when q is even: the probability to have a center is positive, in contrast with what happens in the linear case, or also when q is odd, where this probability is zero.

Mathematics Subject Classification. 37H10, 34F05.

Keywords. Ordinary differential equations with random coefficients, Planar quasi-homogeneous vector fields, Critical point index, Phase portraits.

1. Introduction and Main Results

In this work, a random planar vector field will be a polynomial vector field whose coefficients are normally distributed independent random variables with zero mean and standard deviation one. This is the natural distribution when one is worried about the probability of appearance of some phase portrait for given family of planar vector fields, see for instance [15,16] or [4, Thm 2.1] to have more details. Some related papers that use a similar approach and also study planar random systems are [2,3,17]. In [2] the quadratic systems are considered, in [3] the authors study the homogeneous systems of degree 1,2 and 3 and in [17] the linear systems.

Since we will deal with a class of quasi-homogeneous planar polynomial vector fields, in next section we will recall some usual notations and definitions and some of their properties. One of the most important to our interests is

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