SINGULAR LIMIT CYCLES FOR A CLASS OF DIFFERENTIAL EQUATIONS

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Abstract. In this paper we prove that the maximum number of singular limit cycles for a class of planar differential equation with discontinuous righthand sides, when there are finitely many, is four. Furthermore, we show that there are at most two of these singular limit cycles surrounding the origin.

1. Definitions and main results.

In this paper we study the number of limit cycles of the differential equation with discontinuous righthand sides

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

where $(P,Q) = (P^-,Q^-)$ in $\{y \le 0\}$, $(P,Q) = (P^+,Q^+)$ in $\{y \ge 0\}$, $(P^-,Q^-) = Y_p + Y_q$, $p < q$, and $(P^+,Q^+) = X_n + X_m$, $n < m$. Here, Y_u and X_u are homogeneous polynomial vector fields of degree u. We denote the x-axis by $L = \{(x,y) : y = 0\}$ and the open upper (lower) half plane by $G^+(G^-)$. Note that L is possibly a discontinuous line for system (1). From now on, we will assume that near the origin the flow of the systems $(\dot{x},\dot{y}) = (P^\pm,Q^\pm)$ circle it counterclockwise, that is, $b^-(\theta) = \langle(-\sin\theta,\cos\theta), Y_p(\cos\theta,\sin\theta)\rangle > 0$, and $b^+(\theta) = \langle(-\sin\theta,\cos\theta), X_n(\cos\theta,\sin\theta)\rangle > 0$.

System (1) covers a wide range of physical phenomena, see [AKV]. For these kind of differential equations the concept of solution needs to be re-examined and re-defined in this setting. We will follow the approach of [F].

First of all, note that, on L, Q^+ has the expression $Q^+(x,0) = q_1x^n + q_2x^m$, for $q_1, q_2 \in \mathbf{R}$. Hence $Q^+(x,0)$ has, at most, two non zero simple real roots. The same holds for Q^- on L. As we will see, these roots will play a very important rol in the definition of the solutions of system (1).

Given $p = (c,\tilde{c}) \in \mathbf{R}^2$, we will say that $\gamma(t) = \gamma(t,p)$, is a local solution, for $t \ge 0$, of the Cauchy problem (1) with $\gamma(0) = p$, if one of the following assertions hold:

(i) When $p \in \mathbf{R}^2 \backslash L$, $\gamma(t)$ is the unique local solution of system (1) such that $\gamma(0) = p$.

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