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Uniqueness of Limit Cycles for a Class of Lienard Systems with Applications

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We shall give two criteria for the uniqueness of limit cycles of systems of Liénard type $\dot{x} = -g(y) - f(y)x$, $\dot{y} = h(x)$. We apply them to some polynomial differential equations.

1. TWO CRITERIA OF UNIQUENESS OF LIMIT CYCLES

Consider the system

$$\dot{x} = -g(y) - f(y)x, \qquad \dot{y} = h(x) \tag{1}$$

and define $F(y) = \int_0^y f(u)\,du$.

Assume that the following conditions hold in the region formed by the points (x, y) such that $x \in (-\infty, \infty)$ and $y \in (a, b)$ with $-\infty \leqslant a < 0$ and $0 < b \leqslant +\infty$:

(i) $yg(y) > 0$ for $y \neq 0$, $xh(x) > 0$ for $x \neq 0$;

(ii) $f(y)$, $g(y)$, and $h(x)$ are continuously differentiable, $h(x)$ is increasing, $h(0) = g(0) = 0$, $g'(0) > 0$, and $f(0) < 0$.

We remark that system (1) is the classical Liénard differential equation $\ddot{y} + f(y)\dot{y} + g(y) = 0$ when $h(x) = x$.

Note that from (i) and (ii) the origin is the unique critical point of system (1) and that it is an unstable focus or node.

A *limit cycle* γ is an isolated periodic orbit. A limit cycle is called *stable* (resp. *unstable*) if it is the ω-limit set (resp. α-limit set) of all points in a neighborhood of γ.

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