Publicacions Matemàtiques, Vol 32 (1988), 199–259.

QUADRATIC SYSTEMS WITH A UNIQUE FINITE REST POINT

B. COLL, A. GASULL, J. LLIBRE

Abstract

We study phase portraits of quadratic systems with a unique finite singularity. We prove that there are 111 different phase portraits without limit cycles and that 13 of them are realizable with exactly one limit cycle. In order to finish completely our study two problems remain open: the realization of one topologically possible phase portrait, and to determine the exact number of limit cycles for a subclass of these systems.

0. Introduction and statement of the main results

We consider the differential system $\dot{x} = dx/dt = P(x, y)$, $\dot{y} = dy/dt = Q(x, y)$ where P and Q are polynomials of second degree with real constant coefficients, and x, y, t are also real. We call such systems *quadratic systems*, QS, for short. We assume that these systems have a unique finite singularity, and we denote them by $QS1$.

Our goal is to give all the possible phase portraits (modulus homeomorphisms and changes of the scale of the independent variable t) of the $QS1$ on the sphere of Poincaré (see [G] and [S]). Note that in this study we must take into account the number of limit cycles that the $QS1$ can have. This last problem is the most important difference between this classification and other similar works, see for instance [GLL], [Re].

We prove that there are 111 different phase portraits for $QS1$ without limit cycles. Furthermore 13 of them are also realizables by $QS1$ that have exactly one limit cycle; the phase portraits e_5, e_7, e_8 and e_9 are determinated modulo their number of limit cycles (see Figure 5.16) and we do not know if the phase portrait e_{13} of the same figure exists for some $QS1$. More specifically, to finish completely our study, two problems have resisted our analysis:

(P1) Determine the maximum number of limit cycles that the $QS1$ $\dot{x} = y + px^2 + xy$, $\dot{y} = -x + by + (\ell + bp)x^2 + (m + b)xy$ has with conditions $p > 0$, $(1 + \ell)^2 - 4pm < 0$ and $(m + b - p)^2 - 4(\ell + bp) > 0$.

(P2) Determine if there is some $QS1$ that has phase portrait e_{13} of Figure 5.16.