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Dynamics of a Two Prey and One Predator System with Indirect Effect

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Abstract: We study a population model with two preys and one predator, considering a Holling type II functional response for the interaction between first prey and predator and taking into account indirect effect of predation. We perform the stability analysis of equilibria and study the possibility of Hopf bifurcation. We also include a detailed discussion on the problem of persistence. Several numerical simulations are provided in order to illustrate the theoretical results of the paper.

Keywords: predator–prey; Hopf bifurcation; indirect effects of predation

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1. Introduction

Population dynamics has been extensively studied by researchers in biomathematics, particularly the predator–prey models.

In [1], the authors consider the following two prey one predator model:

$$\begin{aligned} \dot{x} &= rx\left(1-\frac{x}{k}\right)-\frac{cxz}{a+\alpha\eta y+x}, \\ \dot{y} &= y(\beta-\delta z), \\ \dot{z} &= \frac{bxz}{a+\alpha\eta y+x}+\gamma yz-mz, \end{aligned} \tag{1}$$

where x, y, z represent the population densities of the two preys and of the predator, respectively.

In the previous model, for the interaction between the first prey and the predator, they considered a Holling type II functional response where the handling time of predator for the second prey is also involved, whereas for the interaction between the second prey and the predator, they considered a Lotka–Volterra functional response. It is also assumed that there is no intraspecific interaction in the second prey population and its growth is exponential; as a consequence, there is a huge availability of second prey in the absence of a predator, and there is no searching time for the second prey population. They found necessary and sufficient conditions for existence and stability of the nontrivial equilibrium E^* (see [1]).

In order to recover a more complex behavior, we consider a modification of the model that takes the indirect effects of predations into account.

The role played by indirect effects in population dynamics has been investigated in the last several decades (see [2–12]). In the case of predation, it has been pointed out (see [13]) that predator can alter the morphology (see [6]) or the behavior of the preys. In particular, the preys, in order to avoid contacts with predators, may reduce their normal activity or may stay hidden most of the time. Many kinds of indirect effects have been described in the literature (see, for example, [8] for a detailed discussion); an interesting example (see [14]) is the case of refuge indirect effect.