1. INTRODUCTION AND STATEMENT OF THE RESULTS.

We consider the differential system

$$\dot{x} = \frac{dx}{dt} = P(x,y) \quad , \quad \dot{y} = \frac{dy}{dt} = Q(x,y)$$

where P and Q are the polynomial of second degree with real constant coefficients, and x, y, t are also real. We assume that these systems have a unique finite singularity. Such systems will be referred to as quadratic system with a unique singularity, or QS1, for abreviation. For a survey on QS see Coppel [C], Chicone-Jinghuang [CJ] and Ye Yanqian [Ye1] and [Ye2].

Let $X(x,y) = (P(x,y),Q(x,y))$ and suppose that the origin is an isolated singularity. Then, we say that (0,0) is a singularity of type:

E if the determinant of the linear part DX(0,0) is not zero,

S if the linear part DX(0,0) has a unique eigenvalue equal to zero

H if the linear part DX(0,0) has the two eigenvalues equal to zero, and DX(0,0) is not zero.

T if the linear part is zero.

In order to study the singularities at infinity of a quadratic system we need the Poincaré compactification (see [G] and [S]). Consider the sphere $S^2=\{y\in\mathbb{R}^3: y_1^2+y_2^2+y_3^2=1\}$, let $q=(0,0,1)$ be the north pole of S^2, and T_qS^2 be the plane $\{y\in\mathbb{R}^3: y_3=1\}$. Let $p^+:T_qS^2\to S^2$ and $p^-:T_qS^2\to S^2$ be the central projections, i.e., $p^+(y)$ (resp. $p^-(y)$) is the intersection of the line joinning y to the origin with the northern (resp. southern) hemisphere of S^2. Let X be a polynomial vector field of degree d on the plane and let $f:S^2\to\mathbb{R}$ be defined by $f(y)=y_3^{d-1}$. Then the vector fields $f\cdot(p^+)_*X = f\cdot Dp^+(X\circ(p^+)^{-1})$ and $f\cdot(p^-)_*X$, extend X to an analytic vector field $p(X)$, on S^2. The equator is invariant by the flow of $p(X)$ and a neighborhood of the equator corresponds to a neighborhood of infinity in $\mathbb{R}^2$.

We shall say that the infinity of $X(x,y)$ will be degenerate if all the points of the equator of S^2 are singularities of $p(x)$ and maximum {degree(P), degree(Q)} =2.

The singularities of $p(x)$ on the equator of S^2 are called singularities at infinity of X.

This paper first establishes necessary and sufficient conditions for a quadratic system have a unique finite singularity, i.e. to be a QS1 (sec 2). Then determines all the phase portraits for such QS1 with has either the infinity degenerate (sec 3), or same singularity of type T at infinity (sec 4). Also, determines all the possible phase portraits for such QS1 with has a singularity of type H at infinity without take into account the existence and number of limit cycles (sec 5). All the phase portrait in the above hypotheses are given in Figure 1.

Agraiments

Vull agrair al Dr. Jaume LLibre la bona disposició i l'interés mostrat en la direcció d'aquesta tesina. També no vull passar per alt la valuosa col.laboració de l'Armengol Gasull, i en general l'agradable ambient de treball del Departament de Matemàtiques de la Facultat d'Econòmiques.

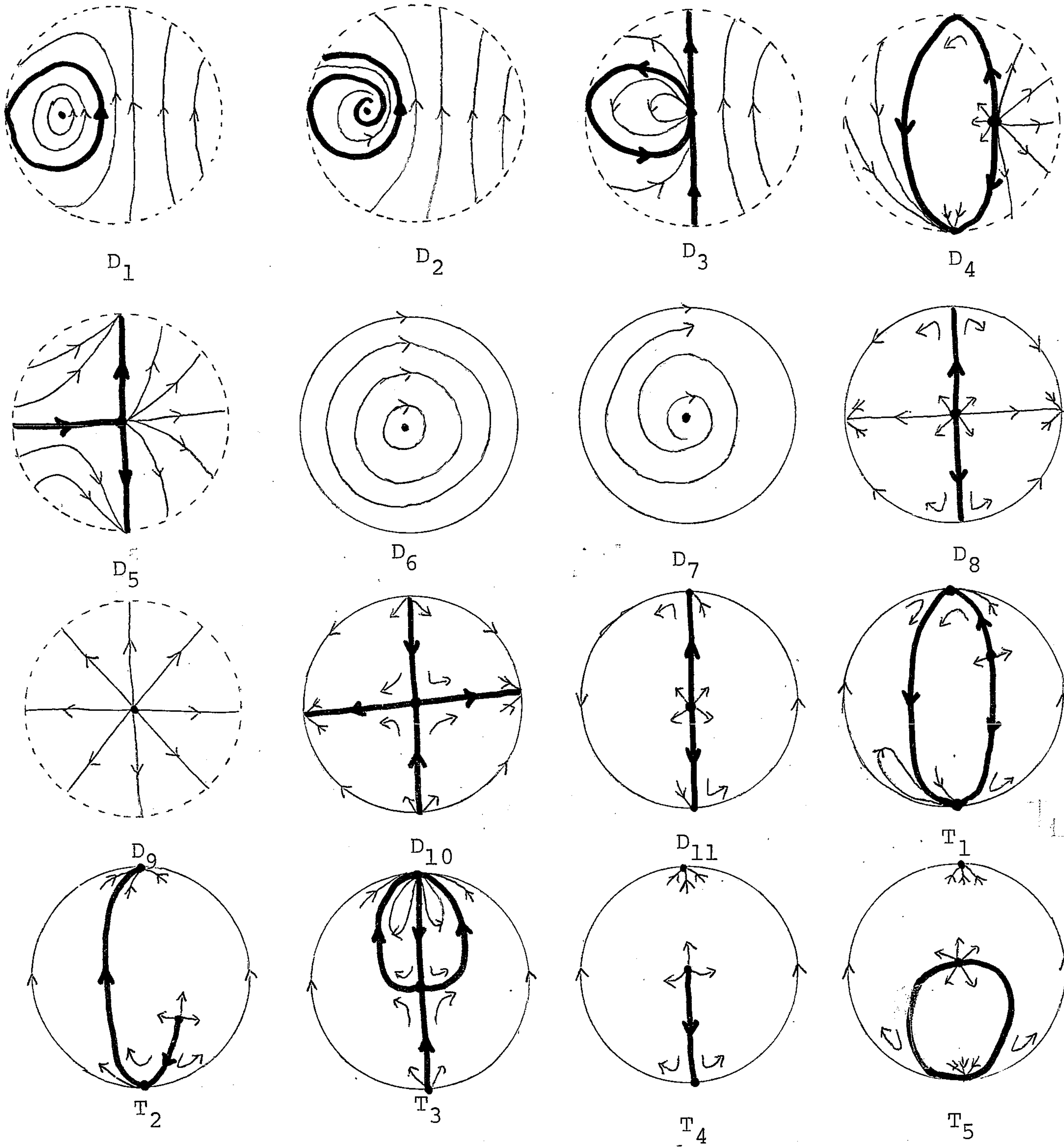
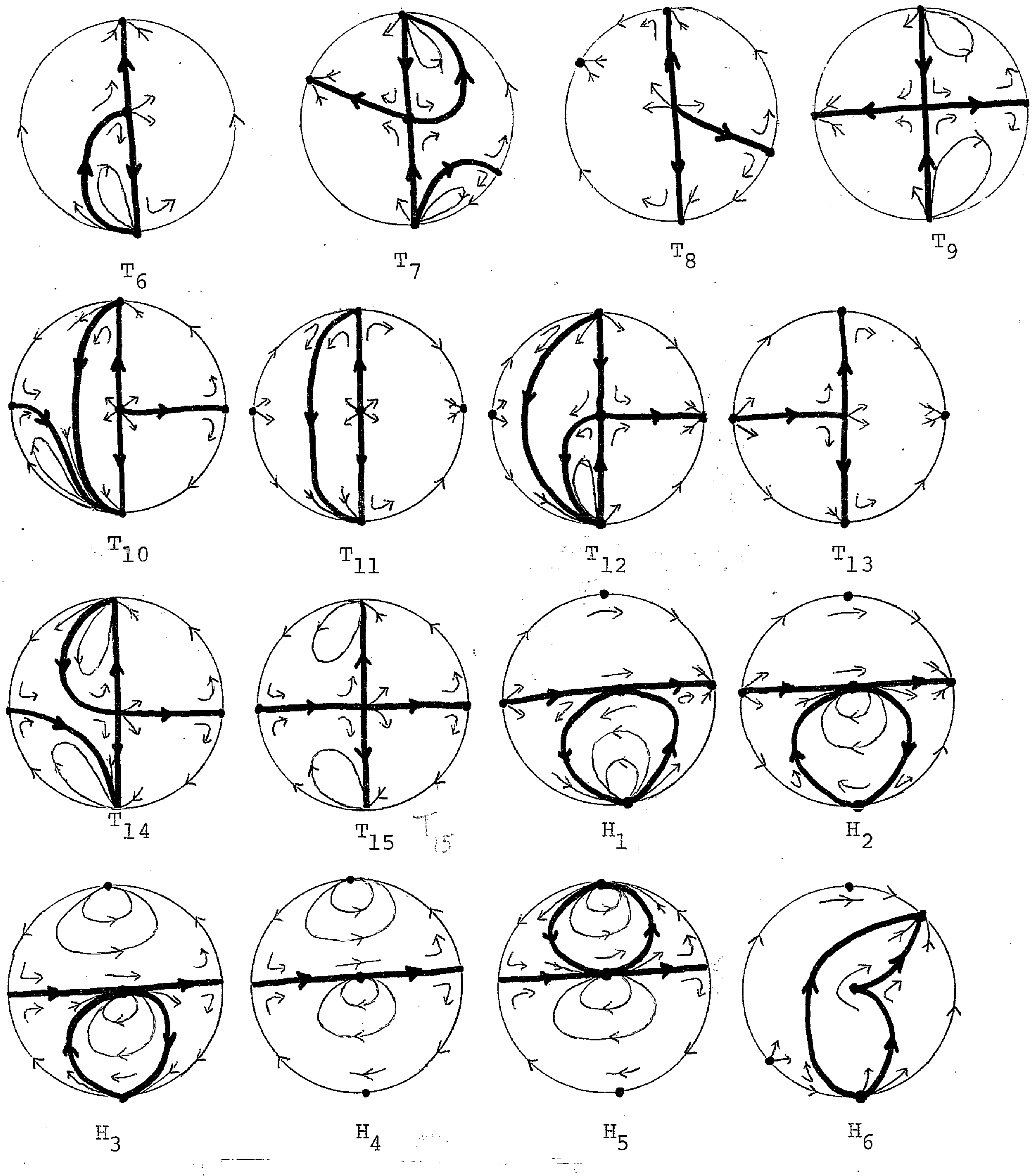


Figure 1.



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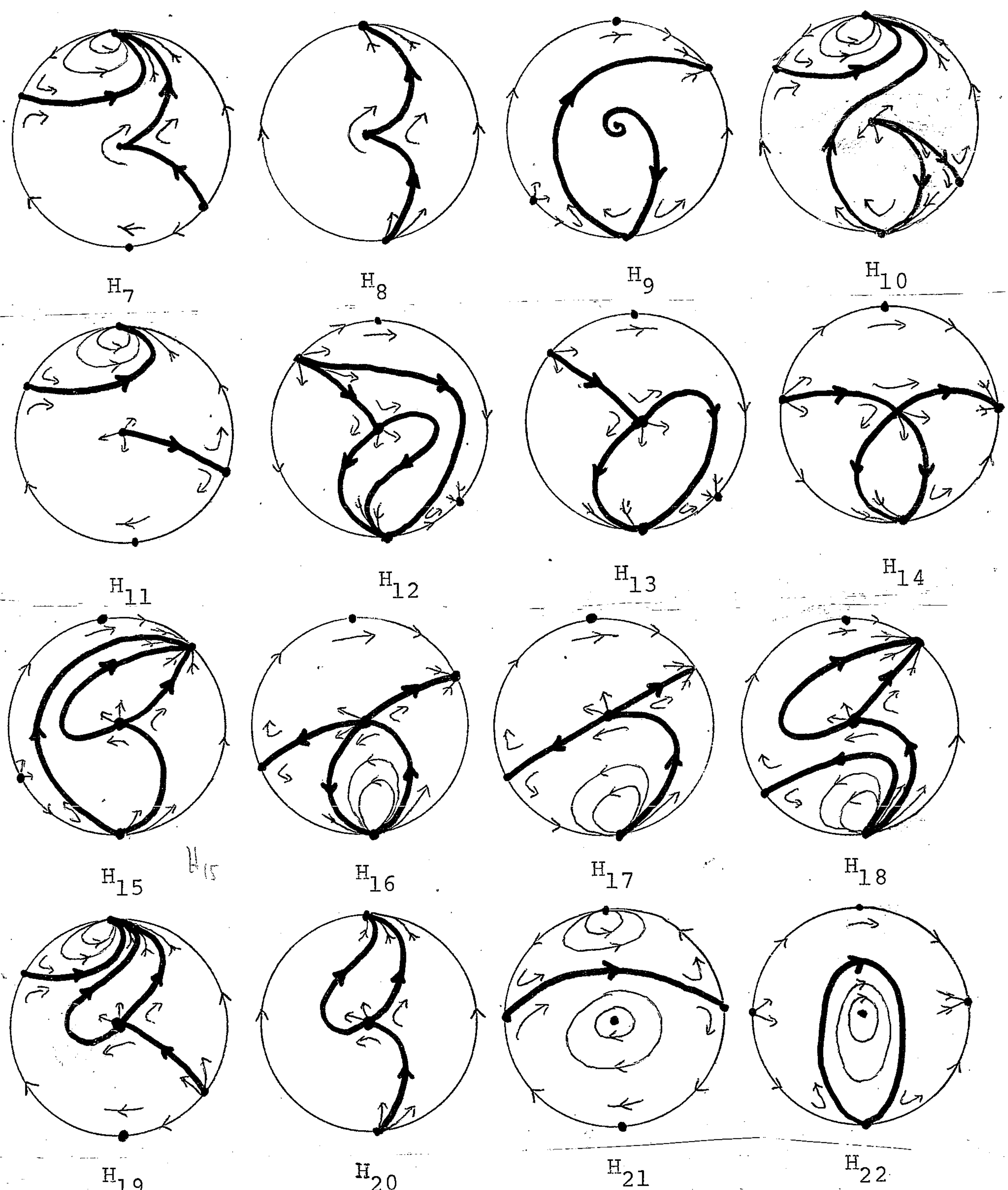

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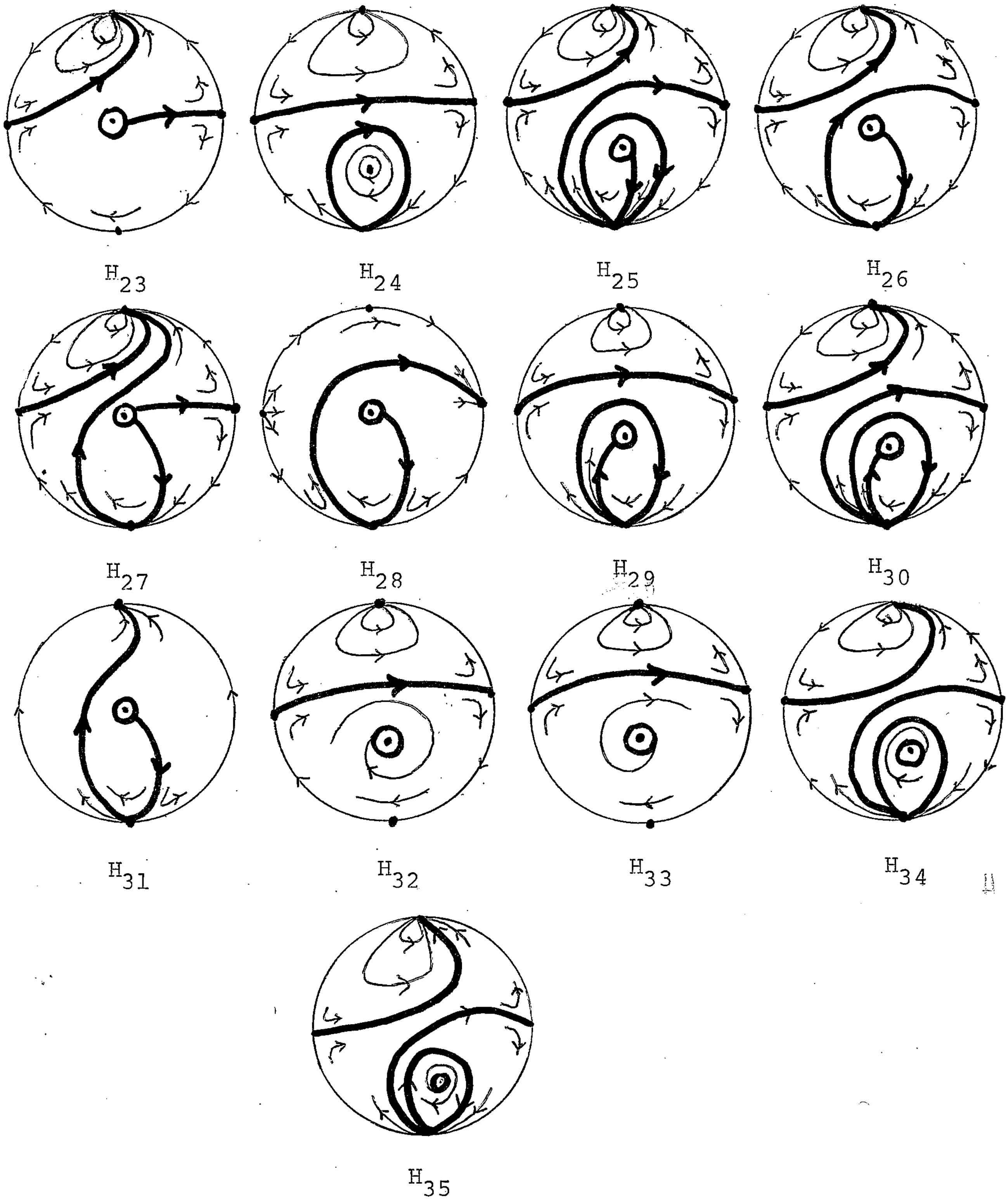

Figure 1. The simbol →⊙ denotes either a stable node or focus or stable or unstable focus on the interior of one or more limit cycles, the outermost of with is externally stable. The symbol ←⊙ is similarly defined.