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Algebraic and Topological Classification of the Homogeneous Cubic Vector Fields in the Plane

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INTRODUCTION

Tsutomu Date and Masao Iri in [DI] gave an algebraic classification of systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where P and Q are homogeneous polynomials of degree 2. For this, they used the classification of the binary cubic forms and also the simultaneous classification of a linear binary form and a cubic binary form given by the algebraic invariant theory.

We begin by doing a similar study for systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where P and Q are homogeneous polynomials of degree three (i.e., *cubic systems*). The classification's theorem of such systems is based on the classification of fourth-order binary forms. Gurevich in [Gu] did the classification of fourth-order binary forms on the field of complex numbers. Since we did not find the classification on the real domain, we adapt Gurevich's proof to obtain it. In Section 1 we give some definitions and preliminary results, while in Section 2 we give the theorem of classification of fourth-order binary forms on the real domain. The method used in the proof (Caley's method) let us obtain canonical forms of the fourth-order binary forms and the algebraic characteristics. Section 3 is devoted to obtaining the algebraic classification of systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where P and Q are homogeneous polynomials of degree 3. Given an arbitrary system $X = (P, Q)$ with P and Q homogeneous polynomials of degree 3, we can know the equivalence-class at which it belongs through the algebraic characteristics.

In Section 4 we study the phase-portraits of systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where P and Q are homogeneous polynomials of degree n and P and Q have no common factor. Such systems have been studied by J. Argemi in [A]. Here we give a shorter new proof of his results by using

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