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Stability index of linear random dynamical systems

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Abstract. Given a homogeneous linear discrete or continuous dynamical system, its stability index is given by the dimension of the stable manifold of the zero solution. In particular, for the n dimensional case, the zero solution is globally asymptotically stable if and only if this stability index is n. Fixed n, let X be the random variable that assigns to each linear random dynamical system its stability index, and let p_k with $k = 0, 1, \ldots, n$, denote the probabilities that $P(X = k)$. In this paper we obtain either the exact values p_k, or their estimations by combining the Monte Carlo method with a least square approach that uses some affine relations among the values $p_k, k = 0, 1, \ldots, n$. The particular case of n-order homogeneous linear random differential or difference equations is also studied in detail.

Keywords: stability index, random differential equations, random difference equations, random dynamical systems.

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1 Introduction

Nowadays it is unnecessary to emphasize the importance of ordinary differential equations and discrete dynamical systems to model real world phenomena, from physics to biology, from economics to sociology. These dynamical systems, a concept that includes both continuous and discrete models (and even dynamic equations in time-scales), can have undetermined coefficients that in the case of real applications must be adjusted to fixed values that serve to make good predictions: this is known as the identification process. Once these coefficients are fixed we obtain a deterministic model.

In recent years some authors have highlighted the utility of considering random rather than deterministic coefficients to incorporate effects due to errors in the identification process,

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