



Phase Portraits of Random Planar Homogeneous Vector Fields

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Received: 30 July 2020 / Accepted: 7 December 2020

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Abstract

In this paper, we study the probability of occurrence of phase portraits in the set of random planar homogeneous polynomial vector fields, of degree n . In particular, for $n = 1, 2, 3$, we give the complete solution of the problem; that is, we either give the exact value of each probability of occurrence or we estimate it by using the Monte Carlo method. Remarkably is that all but two of these phase portraits are characterized by the index at the origin and by the number of invariant straight lines through this point.

Keywords Ordinary differential equations with random coefficients · Planar homogeneous vector fields · Index · Phase portraits

Mathematics Subject Classification 37H10 · 34F05

The authors are supported by Ministry of Science and Innovation–State Research Agency of the Spanish Government through Grants PID2019-104658GB-I00 (MICINN/AEI, Anna Cima and Armengol Gasull) and DPI2016-77407-P (MICINN/AEI/FEDER, UE, Víctor Mañosa). The first and second authors are also supported by the Grant 2017-SGR-1617 from AGAUR, Generalitat de Catalunya. The third author acknowledges the group’s research recognition 2017-SGR-388 from AGAUR, Generalitat de Catalunya.

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