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A Note on the Lyapunov and Period Constants

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Abstract
It is well known that the number of small amplitude limit cycles that can bifurcate from the origin of a weak focus or a non degenerated center for a family of planar polynomial vector fields is governed by the structure of the so called Lyapunov constants, that are polynomials in the parameters of the system. These constants are essentially the coefficients of the odd terms of the Taylor development at zero of the displacement map. Although many authors use that the coefficients of the even terms of this map belong to the ideal generated by the previous odd terms, we have not found a proof in the literature. In this paper we present a simple proof of this fact based on a general property of the composition of one-dimensional analytic reversing orientation diffeomorphisms with themselves. We also prove similar results for the period constants. These facts, together with some classical tools like the Weirstrass preparation theorem, or the theory of extended Chebyshev systems, are used to revisit some classical results on cyclicity and criticality for polynomial families of planar differential equations.

Keywords Limit cycle · Critical period · Bifurcation · Lyapunov and period constants · Cyclicity · Criticality

Mathematics Subject Classification Primary 34C23; Secondary 34C07 · 37G15

1 Introduction and Main Results

Consider planar analytic vector fields $(x, y) \to F(x, y, \lambda) \in \mathbb{R}^2$ with $\lambda \in \mathbb{R}^m$ that have $(x, y) \to (-y, x)$ as their linearization at the origin. It is well known that for

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