



The Limit Cycles of the Higgins–Selkov Systems

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Abstract

In this paper, we investigate the problem of limit cycles for general Higgins–Selkov systems with degree $n + 1$. In particular, we first prove the uniqueness of limit cycles for a general Liénard system, which allows for discontinuity. Then, by changing the Higgins–Selkov systems into Liénard systems, theorems and some techniques for Liénard systems can be applied. After, we prove the nonexistence of limit cycles if the bifurcation parameter is outside an open interval. Finally, we complete the analysis of limit cycles for the Higgins–Selkov systems showing its uniqueness.

Keywords Higgins–Selkov system · Liénard system of arbitrary degree · Uniqueness of limit cycles · Nonexistence of Limit cycles

Mathematics Subject Classification Primary 34C07 · 34C23 · 49J52

1 Introduction and Main Results

In the qualitative theory of planar polynomial differential systems, it is well known how difficult is to study the famous Hilbert’s 16th problem, see Ilyashenko (2002), Li

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