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A note on the existence of invariant punctured tori in the planar circular restricted three-body problem

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Abstract. The existence of transversal ejection–collision orbits in the restricted three-body problem is shown to imply, via the KAM theorem, the existence, for certain intervals of (large) values of the Jacobi constant, of an uncountable number of invariant punctured tori in the corresponding (non-compact) energy surface. The proof is based on a comparison between Levi-Civita and McGehee regularizing variables. That these transversal ejection–collision orbits do actually exist was proved in [**5**] in the case where one of the primaries has a small mass and the zero-mass body revolves around the other (and for all values of the Jacobi constant compatible with the existence of three connected components for the Hill region); it is proved here without any restriction on the masses, well in the spirit of Conley's thesis [**3**].

Part of our setting and notations are collected in figure 1; they are essentially those of [**2**], [**3**] and [**5**].

The position of the zero-mass body in the moving frame is given by the complex number x; in the variables x, $y = dx/dt + i\omega x$, (one half of) the Jacobi constant becomes

$$H(x, y) = |y|^2 + i(\bar{x}y - x\bar{y}) - 2\nu/|x| - 2\mu/|x+1| - \mu(x+\bar{x}) + 2\mu,$$

and Newton's equations read

$$dx/dt = \partial H/\partial \bar{y}, \qquad dy/dt = -\partial H/\partial \bar{x}.$$

(We have used the usual normalization $G = 1$, $\mu + \nu = 1$, which implies $\omega = 1$, and chosen the same constant 2μ as Conley.)

The value $-1/\varepsilon^2$ of H (which is taken large and negative) defines our main parameter ε. If ε is small enough, there are three Hill regions and we shall suppose (as Conley does) that the zero-mass body rotates around the primary of mass ν. Notice that the limit situation $\varepsilon = 0$ corresponds to a collision.† Fixing ε, we regularize this collision (as Conley did) using Levi-Civita variables

$$x = 2z^2, \qquad y = w/\varepsilon\bar{z}, \qquad dt = 2\varepsilon|x|\, dt'$$

† For a general view of collision problems see [**6**].