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Phase portraits and bifurcation diagram of the Gray-Scott model

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ABSTRACT

We give a complete classification of the phase portraits in the Poincaré disk for the cubic polynomial systems

$$\dot{x} = 1 - x - axy^2, \qquad \dot{y} = -by + axy^2,$$

in $\mathbb{R}^2$ according with the values of its two parameters a and b. These differential systems correspond to the Gray-Scott model. Moreover we provide the bifurcation diagram in the parameter plane (a, b) of these systems.

1. Introduction and statement of the main results

The Gray-Scott model [14,20] is a cubic autocatalysis system that exhibits many interesting patterns, see for instance the papers [3,11,18,20] and the references quoted there. This model has been studied with slightly distinct differential equations, here we consider the model given by the differential equations

$$\begin{aligned} \frac{\partial U}{\partial t} &= 1 - U - aUV^2 + D_U \Delta U, \\ \frac{\partial V}{\partial t} &= -bV + aUV^2 + D_V \Delta V, \end{aligned} \tag{1}$$

where $U = U(u, v, t)$ and $V = V(u, v, t)$ are the concentrations of an inhibitor and an activator, $(u, v) \in \mathbb{R}^2$, a and b are positive constants, $D_U > D_V$ are the diffusivities, Δ is the Laplace operator, and t is the time. For more details on the Gray-Scott model see the previous mentioned references.

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