JGP - Vol. 6, n. 3, 1989

Symmetric central configurations of the spatial n-body problem

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Abstract. *We characterize the non-planar central configurations of the spatial n-body problem with equal masses which are orbits of a finite group of isometries of $\mathbf{R}^3$. As a corollary we obtain that the spatial n-body problem with equal masses and $n > 5$ has at least two equivalence classes of non-planar central configurations modulo homotheties and rotations.*

1. INTRODUCTION AND STATEMENT OF THE RESULTS

Let $q_1, \dots, q_n \in \mathbf{R}^d$ denote the positions of n bodies with masses $m_1, \dots, m_n$ respectively. Their motion is described by the equations

$$m_i \ddot{q}_i = -\sum_{\substack{j=1 \\ j \neq i}}^{n} m_i m_j \frac{q_i - q_j}{|q_i - q_j|^3} = -\nabla_i V(q) \qquad \text{for} \quad i = 1, \dots, n; \tag{1.1}$$

where

$$V(q) = -\sum_{i<j} \frac{m_i m_j}{|q_i - q_j|}$$

is the potential energy $V : \mathbf{R}^{dn} \setminus \Delta \to \mathbf{R}$ and $\Delta = \bigcup_{i<j} \Delta_{ij}$ is the set of collisions because $\Delta_{ij} = \{(q_1, \dots, q_n) \in \mathbf{R}^{dn} \mid q_i = q_j\}$. We fix the center of mass $\sum m_i q_i$

Key-Words: n-body problem, symmetries.
1980 MSC: 70 F 10, 20 H 15.