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ALGEBRAIC PROPERTIES OF THE LEFSCHETZ ZETA FUNCTION, PERIODIC POINTS AND TOPOLOGICAL ENTROPY

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Abstract

The Lefschetz zeta function associated to a continuous self-map f of a compact manifold is a rational function P/Q. According to the parity of the degrees of the polinomials P and Q, we analize when the set of periodic points of f is infinite and when the topological entropy is positive.

1. Introduction

In dynamical systems, it is often the case that algebraic information can be used to study qualitative and quantitative properties of the system. In this paper we study the dynamical consequences of simple algebraic properties of the Lefschetz zeta function $Z_f(t)$ associated to a continuous self-map $f : M \to M$ of a compact manifold M, which is always a rational function, i.e. $Z_f(t) = P(t)/Q(t)$ where $P(t)$ and $Q(t)$ are polynomials. We show that there is a relation between the parity of the degrees of $P(t)$ and $Q(t)$ and the finiteness of the set of periodic points of f on one hand, and vanishing topological entropy on the other.

In Section 2 we shall review the definition and some basic properties of the Lefschetz zeta function associated to a self-map and, in particular, give sufficient conditions for $P(t)$ and $Q(t)$ to have a finite factorization in cyclotomic polynomials. The statements and proofs of the results are given in Section 3.

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