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Qualitative analysis of the anisotropic Kepler problem

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ABSTRACT. We give a qualitative analysis of the flow of the anisotropic Kepler problem described by the Hamiltonian system:

$$\dot{q} = M^{-1}p\ , \quad \dot{p} = -q/\|q\|^3 \qquad (1)$$

where $(q,p) \in (R^2/\{0\})\times R^2$, $M^{-1} = \begin{pmatrix} \mu & 0 \\ 0 & 1 \end{pmatrix}$ is the mass matrix and the parameter μ belongs to $[1,\infty)$. It was introduced by Gutzwiller and later it was studied by Devaney. When $\mu = 1$ it is the Kepler problem (an integrable system) and we show the global orbit structure by taking into account the blow up of the singularities at $q = 0$ and $\|q\| = \infty$. When $\mu \in (9/8,\infty)$ symbolic dynamic allows us to classify the solutions of (1). In fact, we prove that the dynamic behaviour contains a subshift with an infinite alphabet. The symbols of this alphabet takes into account the symmetries. For each periodic sequence of this subshift we show the existence of a symmetric periodic orbit which realizes it. The transition from $\mu = 1$ (integrable) to $\mu > 9/8$ (chaotic) is such that the chaos does not appear until $\mu = 9/8$.

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