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CHORDAL CUBIC SYSTEMS

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Abstract

We classify the phase portraits of the cubic systems in the plane such that they do not have finite critical points, and the critical points on the equator of the Poincaré sphere are isolated and have linear part non-identically zero.

1. Introduction

We consider *cubic systems* (CS, for abbreviation), i.e. two–dimensional autonomous systems of differential equations of the form

$$\dot{x} = P(x,y) \quad , \quad \dot{y} = Q(x,y), \tag{1.1}$$

where P and Q are real polynomials such that max {degree P, degree Q} $= 3$. If a CS has no finite critical points, then it will be called *chordal cubic system*. We shall denote by *CCS* the chordal cubic systems such that they only have isolated critical points on the equator of the Poincaré sphere (see [8], [16] or Appendix A of [6]) and the linear part of these critical points are not identically zero. The chordal systems were studied by Kaplan [10], [11]. The name of chordal system is due to the fact that a such system has all its solutions starting and endding at the equator of the Poincaré sphere.

In this paper we give a classification of the phase portraits (on the Poincaré disk) of CCS. A complete study for the chordal quadratic systems has been done by Gasull, Sheng Li–Ren and Llibre in [7].

Our main result is the following one.

Theorem. *The phase portrait of a CCS is homeomorphic (except, perhaps the orientation) to one of the separatrix configurations shown in Figure 1. Furthermore, all the separatrix configurations of Figure 1 are realizable for chordal cubic systems.*

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