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Limit cycles of a class of polynomial systems

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Synopsis

We study the class of polynomial vector fields of the form $\dot{x} = \alpha x - y + P_n(x, y)$, $\dot{y} = x + \alpha y + Q_n(x, y)$, where P_n and Q_n are homogeneous polynomials of degree n. If we define the functions $f(x, y) = xP_n(x, y) + yQ_n(x, y)$ and $g(x, y) = xQ_n(x, y) - yP_n(x, y)$, we characterise the number of limit cycles for this class when the function $g(\alpha g - f)$ does not change sign.

1. Introduction

We consider two-dimensional autonomous systems of differential equations of the form

$$\left.\begin{aligned} \dot{x} &= \alpha x - y + P_n(x, y), \\ \dot{y} &= x + \alpha y + Q_n(x, y), \end{aligned}\right\} \tag{1.1}$$

where P_n and Q_n are real homogeneous polynomials of degree $n \geqq 2$. It is known that the origin of (1.1) is a focus or a centre.

In polar coordinates, system (1.1) is of the form

$$\left.\begin{aligned} \dot{r} &= \alpha r + r^n f(\theta), \\ \dot{\theta} &= 1 + r^{n-1} g(\theta), \end{aligned}\right\} \tag{1.2}$$

where

$$f(\theta) = \cos\theta P_n(\cos\theta, \sin\theta) + \sin\theta Q_n(\cos\theta, \sin\theta),$$

$$g(\theta) = \cos\theta Q_n(\cos\theta, \sin\theta) - \sin\theta P_n(\cos\theta, \sin\theta).$$

It is known that limit cycles of these systems that do not cut the curve $\dot{\theta} = 0$ can be studied by making the transformation $T(r, \theta) = (p, \theta)$, where

$$p = r^{n-1}/[1 + r^{n-1} g(\theta)]. \tag{1.3}$$

This transformation was used in **[2]**, **[9]**, **[12]**, **[13]**, **[1]** and **[4]**. In the new coordinates (p, θ) the system becomes

$$dp/d\theta = A(\theta)p^3 + B(\theta)p^2 + (n-1)\alpha p, \tag{1.4}$$