Discrete and Continuous Dynamical Systems
Vol. 42, No. 9, September 2022, pp. 4237–4261
doi:10.3934/dcds.2022051

ACHIEVABLE CONNECTIVITIES OF FATOU COMPONENTS FOR A FAMILY OF SINGULAR PERTURBATIONS

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(Communicated by Enrique Pujals)

Abstract. In this paper we study the connectivity of Fatou components for maps in a large family of singular perturbations. We prove that, for some parameters inside the family, the dynamical planes for the corresponding maps present Fatou components of arbitrarily large connectivity and we determine precisely these connectivities. In particular, these results extend the ones obtained in [5, 6].

1. **Introduction.** In the recent decades there has been an increasing interest in studying families of rational maps usually called *singular perturbations*. Roughly speaking, a family is called a singular perturbation if it is defined by a *base* family (called the *unperturbed family* and for which we have a deep understanding of the dynamical plane) plus a *local* perturbation, that is, a perturbation which has a significant effect on the orbits of points in some part(s) of the dynamical plane, but a very small dynamical relevancy on other regions.

Singular perturbations, no matter the concrete formulas, have some common properties which make their study interesting. On the one hand, the degree of the unperturbed family is smaller than the degree of the perturbed one. Consequently, one should expect richer dynamics for singular perturbations than for the unperturbed maps. On the other hand, most of this new freedom arising from the perturbation may be captive of the dynamical properties of the unperturbed family. The balance between these two scenarios has become very successful in finding new dynamical phenomena.

The relation between the topology of the dynamically invariant sets (Fatou and Julia set) and the behaviour of the critical orbit(s) is an important issue when studying the dynamical plane of a particular rational map. A paradigmatic example

2020 *Mathematics Subject Classification.* Primary: 37F10, 37F12, 37F20, 37F44; Secondary: 30D05.

Key words and phrases. Holomorphic dynamics, Fatou and Julia sets, rational maps, singular perturbation, connectivity of Fatou components.

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