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Dynamical mechanism behind ghosts unveiled in a map complexification

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ABSTRACT

Complex systems such as ecosystems, electronic circuits, lasers, or chemical reactions can be modelled by dynamical systems which typically experience bifurcations. It is known that transients become extremely long close to bifurcations, also following well-defined scaling laws as the bifurcation parameter gets closer the bifurcation value. For saddle-node bifurcations, the dynamical mechanism responsible for these delays, tangible at the real numbers phase space (so-called ghosts), occurs at the complex phase space. To study this phenomenon we have complexified an ecological map with a saddle-node bifurcation. We have investigated the complex (as opposed to real) dynamics after this bifurcation, identifying the fundamental mechanism causing such long delays, given by the presence of two repellers in the complex space. Such repellers appear to be extremely close to the real line, thus forming a narrow channel close to the two new fixed points and responsible for the slow passage of the orbits. We analytically provide the relation between the well-known inverse square-root scaling law of transient times and the multipliers of these repellers. We finally prove that the same phenomenon occurs for more general i.e. non-necessarily polynomial, models.

1. Introduction

Bifurcations are responsible for qualitative changes in dynamical systems due to parameter changes [1,2]. Local bifurcations typically involve stability shifts or collisions between fixed points. Classical examples are transcritical, saddle-node (hereafter labeled as s-n, also named fold or tangent), pitchfork, or Hopf-Andronov bifurcations [2]. Bifurcations occur in most physical systems and have been mathematically described in elastic-plastic materials [3], electronic circuits [4,5], or open quantum systems [6], among many others. Bifurcations have been also largely investigated in population dynamics [7–11] since they often involve important changes such as the separation between species' persistence and extinctions. Further theoretical research in socioecological systems [12,13], in autocatalytic systems [14–16], in the fixation of alleles in population genetics and biological or computer virus propagation [17–20], has revealed bifurcation phenomena, often governed by abrupt changes (typically due to s-n bifurcations). Additionally, bifurcations have been identified experimentally in many physical [5,21–23], chemical [24,25], and biological systems [26,27].

One of the most remarkable properties of systems approaching a local bifurcation is that transients' lengths slow down drastically. The length of these transients typically scales with the distance to the bifurcation value [2,28]. Such scaling properties are found both in continuous-time (flows) and discrete-time (maps) dynamical systems. For instance, the length of transients, τ, in transcritical bifurcations diverges as a power law $\tau \sim |\mu - \mu_c|^{-1}$ [28,29], with μ and μ_c being, respectively, the control parameter and the value at which it bifurcates. The same scaling exponent is found in the supercritical Pitchfork bifurcation in flows [28], and in the Pitchfork and the period-doubling bifurcation in maps [29]. For the s-n, transients scale as $\tau \sim |\mu - \mu_c|^{-1/2}$ for flows and maps [2,14,30] (see Fig. 1 and Fig. 7a). Remarkably, this scaling law was found experimentally in an electronic circuit modelling Duffing's oscillator [5]. The same scaling exponent has been recently found in a delayed differential equation suffering a s-n bifurcation [15].

In short, the s-n bifurcation involves the collision and consequent annihilation of fixed points. This annihilation actually involves the jump of these two equilibria from the real numbers

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