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Research paper

Connectivity of the Julia set for the Chebyshev-Halley family on degree n polynomials

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ABSTRACT

We study the Chebyshev-Halley family of root finding algorithms from the point of view of holomorphic dynamics. In this paper we provide a criterion which guarantees the simple connectivity of the basins of attraction of the roots. We use the criterion for the Chebyshev-Halley methods applied to the degree n polynomials $z^n + c$, obtaining a characterization of the parameters for which all Fatou components are simply connected and, therefore, the Julia set is connected. We also study how increasing n affects the dynamics.

1. Introduction

Most of the problems faced by scientists and engineers involve equations that do not have a known analytical solution. Numerical methods are a good option to tackle and solve real world problems. In particular, iterative methods are used to find approximations of the solutions of $f(z) = 0$.

The best-known root-finding algorithm is Newton's method, which has order of convergence 2. Many numerical methods of order three or more are derived from Newton's scheme: Chebyshev method, also known as super-Newton method (see [15], for example), Halley's method and super-Halley method. A more detailed study of the construction and evolution of these numerical methods can be seen in [13]. These methods belong to a family of numerical algorithms called the Chebyshev-Halley family, which is given by

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}\frac{L_f(x_n)}{1 - \alpha L_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{1}$$

where

$$L_f(x_n) = \frac{f(x_n)f''(x_n)}{(f'(x_n))^2}$$

and $\alpha \in \mathbb{C}$. Within this family, Chebyshev method is obtained for $\alpha = 0$, Halley's method is obtained for $\alpha = \frac{1}{2}$ and super-Halley method is obtained for $\alpha = 1$. Moreover, as α tends to ∞ these algorithms converge to Newton's method.

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