



Centers and Limit Cycles of Vector Fields Defined on Invariant Spheres

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Abstract

The aim of this paper is the study of the center-focus and cyclicity problems inside the class \mathcal{X} of 3-dimensional vector fields that admit a first integral that leaves invariant any sphere centered at the origin. We classify the centers of linear, quadratic homogeneous and a family of quadratic vector fields $\mathcal{F} \subset \mathcal{X}$, restricted to one of these spheres. Moreover, we show the existence of at least 4 limit cycles in family \mathcal{F} .

Keywords Vector fields on invariant spheres · Integrability · Center-focus problem · Local cyclicity

Mathematics Subject Classification Primary 34C07; Secondary 34C23 · 37C27

1 Introduction

Differential equations and dynamical systems appear naturally in the description of many phenomena for which local processes are known. The central problem is then to obtain global information on these phenomena. Once the local equations are formu-

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