



# Phase portraits of (2;0) reversible vector fields with symmetrical singularities



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## ABSTRACT

In this paper we study the phase portraits in the Poincaré disk of the reversible vector fields of type (2;0) having generic bifurcations around a symmetric singular point  $p$ . We also prove the nonexistence of any periodic orbit surrounding  $p$ . We point out that some numerical computations were necessary in order to control the number of limit cycles.

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## 1. Introduction and main results

Given two real  $C^k$ ,  $k \geq 1$ , functions of two variables  $P, Q : \mathbb{R}^2 \rightarrow \mathbb{R}$  we define a *planar  $C^k$  differential system* as a system of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the dot in system (1) denotes the derivative with respect to the independent variable  $t$ . We call the map  $X = (P, Q)$  a *vector field*. If  $P$  and  $Q$  are polynomials then system (1) is a *planar polynomial differential system*. In this case we say that system (1) has *degree  $n$*  if the maximum of the degrees of  $P$  and  $Q$  is  $n$ . If  $n = 1$  then system (1) is called a *linear differential system*. This last class of systems is already completely understood (see for instance chapter 1 of [17]). However for  $n \geq 2$ , that is for nonlinear differential polynomial systems we know very few things. The class of planar polynomial systems with degree  $n \geq 2$  is too wide, so is common to study more specific subclasses and to classify their topological phase portraits. See for instance [1], [10] and [23].

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