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Periodic orbits of a Hamiltonian system related with the Friedmann–Robertson–Walker system in rotating coordinates

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ABSTRACT

We provide sufficient conditions on the four parameters of a Hamiltonian system, related with the Friedmann–Robertson–Walker Hamiltonian system in a rotating reference frame, which guarantee the existence of 12 continuous families of periodic orbits, parameterized by the values of the Hamiltonian, which born at the equilibrium point localized at the origin of coordinates. The main tool for finding analytically these families of periodic orbits is the averaging theory for computing periodic orbits adapted to the Hamiltonian systems. The technique here used can be applied to arbitrary Hamiltonian systems.

1. Introduction and statement of the main results

In astrophysics the study of the dynamics of the universe is an area where the application of the techniques of the dynamical systems provide good results, mainly in galactic dynamics, see the articles [1–5] and the references cited therein.

Recently chaotic motion has been detected in the following simplified version of the Friedmann–Robertson–Walker Hamiltonian

$$H = \frac{1}{2}(p_Y^2 - p_X^2) + \frac{1}{2}(Y^2 - X^2) + \frac{b}{2}X^2Y^2, \tag{1}$$

introduced by Calzeta and Hasi in [6]. In fact this model is too simplified in order to be considered realistic, but it is interesting due to its simplicity and for showing the existence of chaos in cosmology, look for more details in [6]. Hawking [7] and Page [8] used analogous models to analyze the relation between the thermodynamic arrow of time and the cosmology.

A large number of potentials in galactic dynamics are of the form $V(x^2, y^2)$, see the article [9] and the previous mentioned articles on galactic dynamics. These potentials show a reflection symmetry with respect to both axes. Then in [10] was studied the following generalized version of the Calzeta–Hasi's model

$$H = \frac{1}{2}(p_Y^2 - p_X^2) + \frac{1}{2}(Y^2 - X^2) + \frac{a}{4}X^4 + \frac{b}{2}X^2Y^2 + \frac{c}{4}Y^4. \tag{2}$$

Following the classical restricted circular three-body problem in which its dynamics is better understand in a rotating frame that in a sideral frame of coordinates, our objective is to study the dynamics of the generalized version of the Calzeta–Hasi's model (2) in rotating coordinates. More precisely, we consider the following generalized version of the Calzeta–Hasi's model in rotating coordinates that itself is a simplified version of the Friedmann–Robertson–Walker Hamiltonian

$$H = \frac{1}{2}\left(y^2 - x^2 + p_y^2 - p_x^2\right) + \frac{1}{4}\left(ax^4 + 2bx^2y^2 + cy^4\right) - \omega\left(xp_y - yp_x\right), \tag{3}$$

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