



The local period function for Hamiltonian systems with applications

Claudio A. Buzzi^a, Yagor Romano Carvalho^{a,*}, Armengol Gasull^{b,c}

^a *Mathematics Department, Universidade Estadual Paulista Julio de Mesquita Filho, 15054-000 São José do Rio Preto, São Paulo, Brazil*

^b *Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08193 Cerdanyola del Vallès, (Barcelona), Spain*

^c *Centre de Recerca Matemàtica, Edifici Cc, Campus de Bellaterra, 08193, Cerdanyola del Vallès, (Barcelona), Spain*

Received 5 August 2020; accepted 26 January 2021

Available online 3 February 2021

Abstract

In the first part of the paper we develop a constructive procedure to obtain the Taylor expansion, in terms of the energy, of the period function for a non-degenerated center of any planar analytic Hamiltonian system. We apply it to several examples, including the whirling pendulum and a cubic Hamiltonian system. The knowledge of this Taylor expansion of the period function for this system is one of the key points to study the number of zeroes of an Abelian integral that controls the number of limit cycles bifurcating from the periodic orbits of a planar Hamiltonian system that is inspired by a physical model on capillarity. Several other classical tools, like for instance Chebyshev systems are applied to study this number of zeroes. The approach introduced can also be applied in other situations.

© 2021 Elsevier Inc. All rights reserved.

MSC: primary 34C08; secondary 34C25, 37G15, 37J45

Keywords: Period function; Limit cycles; Abelian integrals; Extended complete Chebyshev systems; Picard-Fuchs differential equations

* Corresponding author.

E-mail addresses: claudio.buzzi@unesp.br (C.A. Buzzi), yagor.carvalho@unesp.br (Y.R. Carvalho), gasull@mat.uab.cat (A. Gasull).