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ORIGINAL PAPER

The solution of the second part of the 16th Hilbert problem for nine families of discontinuous piecewise differential systems

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Abstract We provide the maximum number of limit cycles of some classes of discontinuous piecewise differential systems formed by two differential systems separated by a straight line, when these differential systems are linear centers or three families of cubic isochronous centers, giving rise to ten different classes of discontinuous piecewise differential systems. These maximum number of limit cycles vary from 0, 1, 2, 3, 5, 7 and 12 depending on the chosen class. For nine of these classes, we prove that the corresponding maximum number of limit cycles are reached. In particular, we have solved the extension of the second part of the 16th Hilbert problem to these classes of discontinuous piecewise differential systems. The main tool used for proving these results is based on the first integrals of the systems which form the discontinuous piecewise differential systems.

Keywords Discontinuous piecewise differential systems · isochronous cubic systems · linear differential centers · limit cycle

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1 Introduction and the statement of the main results

In this paper, we deal with polynomial differential systems in $\mathbb{R}^2$ of the form

$$\frac{dx}{dt} = P(x, y), \qquad \frac{dy}{dt} = Q(x, y), \tag{1}$$

where the degree of the systems is the maximum degree of the polynomials P and Q.

In 1900, David Hilbert [16–18] gave a talk at the International Congress of Mathematicians in Paris, where he provided a list of 23 problems. Only two of the problems of this list remain open, the Riemann Conjecture and the 16th problem. The second part of the 16th Hilbert problem asked for an upper bound for the maximum number of limit cycles of all polynomial differential systems of a given degree and also for the possible configuration of these limit cycles, see also [22,23].

Here we will work with the discontinuous piecewise vector fields

$$X^{\pm} : (\dot{x}, \dot{y}) = (f^{\pm}(x, y), g^{\pm}(x, y)), \tag{2}$$

defined in the half-planes $\Sigma^{\pm} = \{(x, y) \in \mathbb{R}^2 : \pm x > 0\}$. On the straight line $\Sigma = \{x = 0\}$, the differential system is bivaluated. The straight line Σ is the *discontinuity line* when the two vector fields $X^{\pm}$ do