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ORBITALLY SYMMETRIC SYSTEMS WITH APPLICATIONS TO PLANAR CENTERS

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Abstract. We present a generalization of the most usual symmetries in differential equations known as the time-reversibility and the equivariance ones. We check that the typical properties are also valid for the new definition that unifies both. With it, we are able to present new families of planar polynomial vector fields having equilibrium points of center type. Moreover, we provide the highest lower bound for the local cyclicity of an equilibrium point of polynomial vector fields of degree 6, $M(6) \geq 48$.

1. **Introduction.** One of the fundamental properties studied in natural science is the existence of symmetries. They appear usually in many physical models describing classical mechanics. The most important studied symmetry is known as the time-reversible one, being Birkhoff one of the first who used it. See, for example, his works on the restricted three-body problem studied in 1915 ([6]) or the billiard ball problem published in 1927 ([7]). There exists an extensive bibliography on symmetries and their properties in all areas of dynamical systems. See for example the nice survey of Lamb & Roberts published in 1998 ([24]). In particular, they describe how this time-symmetry is useful in mathematics and physics for understanding a big list of phenomena: symmetric periodic orbits, local bifurcations, homoclinic and heteroclinic orbits,...They appear also in other research branches as thermodynamics and quantum mechanics.

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