Bounded polynomial vector fields in $\mathbb{R}^2$ and $\mathbb{R}^n$

Luis Barreira [a], Jaume Llibre [b], Claudia Valls [a,*]

[a] *Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal*
[b] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

Abstract

We characterize the bounded polynomial vector fields in $\mathbb{R}^2$. Additionally we provide a necessary condition but not sufficient which must be satisfied by bounded polynomial vector fields in $\mathbb{R}^n$.
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1. Introduction

Many interesting problems coming from the physical and natural sciences can be modeled by polynomial vector fields in $\mathbb{R}^2$ as for instance the Lotka–Volterra systems, the Blausius equation, the van der Pol equation, ... [6]. But the polynomial vector fields started to be analyzed from a mathematical point of view in the works of Poincaré [15], Hilbert [11], Bendixson [1], Dulac [8], ... Since the general class of polynomial vector fields in $\mathbb{R}^2$ is very difficult to study, many authors put their attention to several subclasses. Here our main objective is to characterize the class of bounded polynomial vector fields in $\mathbb{R}^2$.

Bounded polynomial vector fields already have been studied by several authors. Thus the bounded quadratic vector fields have been studied by Coll, Dickson, Dumortier, Gasull, Herssens,

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* Corresponding author.
E-mail addresses: barreira@math.tecnico.ulisboa.pt (L. Barreira), jllibre@mat.uab.cat (J. Llibre),
cvalls@math.tecnico.ulisboa.pt (C. Valls).