




Convexity and symmetry of central configurations in the five-body problem: Lagrange plus two

E. Barrabés¹ · J. M. Cors² · A. C. Fernandes³  · C. Vidal⁴

Received: 28 January 2021 / Accepted: 5 July 2021 / Published online: 10 July 2021
© The Author(s), under exclusive licence to Springer Nature Switzerland AG 2021

Abstract

We study convexity and symmetry of central configurations in the five body problem when three of the masses are located at the vertices of an equilateral triangle, that we call *Lagrange plus two central configurations*. First, we prove that the two bodies out of the vertices of the triangle cannot be placed on certain lines. Next, we give a geometrical characterization of such configurations in the sense as that of Dziobek, and we describe the admissible regions where the two remaining bodies can be placed. Furthermore, we prove that any Lagrange plus two central configuration is concave. Finally, we show numerically the existence of non-symmetric central configurations of the five body problem.

1 Introduction and main results

Central configurations of the planar n -body problem correspond to configurations $\mathbf{r} = (\mathbf{r}_1, \dots, \mathbf{r}_n)$, $\mathbf{r}_i \in \mathbb{R}^2$, $i = 1, \dots, n$, for positive masses m_1, \dots, m_n moving under the Newtonian gravitational attraction such that the acceleration at each mass point is a constant multiple of the relative position with respect to the center of mass \mathbf{C} :

✉ A. C. Fernandes
acfernandes@unifei.edu.br

¹ Departament d'Informàtica, Matemàtica Aplicada i Estadística, Universitat de Girona, Girona, Spain

² Departament de Matemàtiques, Universitat Politècnica de Catalunya, Manresa, Spain

³ Instituto de Matemática e Computação, Universidade Federal de Itajubá, Itajubá, MG, Brazil

⁴ Grupo de Investigación en Sistemas Dinámicos y Aplicaciones-GISDA, Departamento de Matemática, Universidad del Bío-Bío, Concepción, Chile