Mediterr. J. Math. (2021) 18:167
https://doi.org/10.1007/s00009-021-01809-2

Mediterranean Journal of Mathematics

Three-Dimensional Lotka–Volterra Systems with 3:−1:2-Resonance

Waleed Aziz, Colin Christopher, Jaume Llibre and Chara Pantazi

Abstract. We study the local integrability at the origin of a nine-parameter family of three-dimensional Lotka–Volterra differential systems with (3:− 1:2)-resonance. We give necessary and sufficient conditions on the parameters of the family that guarantee the existence of two independent local first integrals at the origin of coordinates. Additionally, we classify those cases where the origin is linearizable.

Mathematics Subject Classification. Primary 34A05, 34C20; Secondary 34A34, 34C14.

Keywords. Lotka–Volterra, integrability, linearizability, Jacobi multiplier.

1. Introduction and Statement of the Main Results

Lotka–Volterra systems [35,46] were first introduced in population biology to describe the evolution of conflicting species, see for example [37,38]. Over the years, these systems have been used in many branches of sciences like laser physics [28], neural networks [40], plasma physics [29], chemical kinetics [39], and so forth. The dynamical behaviour of these models has been widely studied, see for instance [1,8,14,34,45,49] amongst others. The integrability of some families of Lotka–Voltera systems was considered by several authors, for example, [15,16,18,31,34,36]. Similar techniques have been applied to other families in [5,23], and some general properties of these systems are also studied in [10,11].

In general, for a three-dimensional system, the existence of a first integral is very helpful because it reduces the study of the dynamics from three dimensions to two. Moreover, the existence of two independent first integrals determines completely the trajectories of the system. Hence, the study of the existence of first integrals is an important theme in the qualitative theory of dynamical systems.

Published online: 11 June 2021