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Integrability and linearizability of a family of three-dimensional quadratic systems

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ABSTRACT

We consider a three-dimensional vector field with quadratic nonlinearities and in general none of the axis plane is invariant. For our investigation, we are interesting in the case of $(1 : -2 : 1)$ – resonance at the origin. Hence, we deal with a nine parametric family of quadratic systems and our purpose is to understand the mechanisms of local integrability. By computing some obstructions, knowing as resonant focus quantities, first we present necessary conditions that guarantee the existence of two independent local first integrals at the origin. For this reason Gröbner basis and some other algorithms are employed. Then we examine the cases where the origin is linearizable. Some techniques like existence of invariant surfaces and Jacobi multipliers, Darboux method, properties of linearizable nodes of two dimensional systems and power series arguments are used to prove the sufficiency of the obtained conditions. For a particular three-parametric subfamily, we provide conditions on the parameters to guarantee the non-existence of a polynomial first integral.

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1. Introduction and statement of the main results

The problem of integrability is one of the most difficult problems in the qualitative theory of differential equations. For a three-dimensional system, two independent first integrals are required in order the system to be completely integrable. In this case, the trajectories of the system are completely determined by the two first integrals. To prove the existence or non-existence of a first integral in general is a very hard problem, especially when the system depends on parameters. So, during the years, many technics have been developed relating to first integrals, like Lie symmetries [32], Darboux theory of integrability [13,14], Painlevé analysis [8], Differential Galois Theory [28,35], among many others.

In this work, we deal with the local integrability and linearizability problem at the origin of the three-dimensional systems

$$\dot{x} = P = \lambda x + axy + bxz + cyz,$$
$$\dot{y} = Q = \mu y + dxy + exz + fyz,$$

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