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Dynamics of the Isotropic Star Differential System from the Mathematical and Physical Point of Views

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Abstract: The following differential quadratic polynomial differential system $\frac{dx}{dt} = y - x$, $\frac{dy}{dt} = 2y - \dfrac{y}{\gamma-1}\left(2 - \gamma y - \dfrac{5\gamma-4}{\gamma-1}x\right)$, when the parameter $\gamma \in (1,2]$ models the structure equations of an isotropic star having a linear barotropic equation of state, being $x = m(r)/r$ where $m(r) \geq 0$ is the mass inside the sphere of radius r of the star, $y = 4\pi r^2\rho$ where ρ is the density of the star, and $t = \ln(r/R)$ where R is the radius of the star. First, we classify all the topologically non-equivalent phase portraits in the Poincaré disc of these quadratic polynomial differential systems for all values of the parameter $\gamma \in \mathbb{R} \setminus \{1\}$. Second, using the information of the different phase portraits obtained we classify the possible limit values of $m(r)/r$ and $4\pi r^2\rho$ of an isotropic star when r decreases.

Keywords: isotropic star; polynomial differential equation; phase portrait; Poincaré disc

MSC: 34C05

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1. Introduction and the Main Results

The structure equations of an isotropic star having a linear barotropic equation of state are

$$\dot{x} = y - x = p(x,y),$$
$$\dot{y} = 2y - \frac{y}{\gamma-1}\left(2 - \gamma y - \frac{5\gamma-4}{\gamma-1}x\right) = q(x,y), \tag{1}$$

where the parameter γ varies in the interval $(1,2]$, and the dot denotes the derivative with respect to the variable $t = \ln(r/R)$ being R the radius of the star. Therefore, from the physical point of view, we are interested in the solutions defined in the interval $t \in (-\infty, 0)$. Here $x = m(r)/r$ where $m(r) \geq 0$ is the mass inside the sphere of radius r of the star, $y = 4\pi r^2\rho$ being ρ the density of the star. For more details on this differential system (1) see [1–3], and for additional information on the isotropic stars see [4–8].

The objective of this paper is double. First, we study the phase portraits of the quadratic systems (1) modeling the structure equations of an isotropic star having a linear barotropic equation of state from a mathematical point of view, i.e., for all the values of parameter $\gamma \in \mathbb{R} \setminus \{1\}$ where the system is defined. These phase portraits are described in the Poincaré disc, in this way we control the orbits that escape or come from infinity, see Theorem 1. Second from the different phase portraits obtained, we classify the possible limit values of $m(r)/r$ and $4\pi r^2\rho$ of an isotropic star when r decreases, as far as we know this information on the behavior of the isotropic stars is new, see Theorem 2.

We remark that from the physics point of view and since $x > 0$ and $y > 0$ we are mainly interested in the dynamics of the differential system (1) in the positive quadrant $Q = \{(x,y) \in \mathbb{R}^2 : x > 0, y > 0\}$ of $\mathbb{R}^2$.