



# Invariant conditions for phase portraits of quadratic systems with complex conjugate invariant lines meeting at a finite point

Joan C. Artés<sup>1</sup> · Jaume Llibre<sup>1</sup> · Dana Schlomiuk<sup>2</sup> · Nicolae Vulpe<sup>3</sup>

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## Abstract

The goal of this article is to give invariant necessary and sufficient conditions for a quadratic system, presented in whatever normal form, to have any one of 17 out of the 20 phase portraits of the family of quadratic systems with two complex conjugate invariant lines intersecting at a finite real point. The systems in this family have a maximum of one limit cycle. Among the 17 phase portraits we have two with limit cycles. We also give invariant necessary and sufficient conditions for a system to have one of the three remaining phase portraits, out of which one has a limit cycle and another one a homoclinic loop. In the region  $\mathcal{R}$  determined by these last conditions, due to the presence of systems with a homoclinic loop, an analytic condition, the three phase portraits cannot be separated by algebraic conditions in terms of invariant polynomials. We also give the bifurcation diagram of this family, outside the region  $\mathcal{R}$ , in the twelve parameter space of coefficients of the systems.

**Keywords** Quadratic vector fields · Infinite and finite singularities · Affine invariant polynomials · Poincaré compactification · Topological configuration of singularities · Phase portrait · Limit cycle

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✉ Joan C. Artés  
artes@mat.uab.cat

Jaume Llibre  
jllibre@mat.uab.cat

Dana Schlomiuk  
dasch@dms.umontreal.ca

Nicolae Vulpe  
nvulpe@gmail.com

<sup>1</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

<sup>2</sup> Département de Mathématiques et de Statistiques, Université de Montréal, Montreal, Canada

<sup>3</sup> Vladimir Andrunakievichi Institute of Mathematics and Computer Science, 5 Academiei str, 2028 Chişinău, Moldova