THE SIMPLE PERIODIC ORBITS IN THE UNIMODAL MAPS

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1. PRELIMINARIES

The Šarkovskiĭ's ordering in N (from now on, for definitions see [2]) is: $3\Delta 5\Delta 7\Delta \ldots \Delta 2.3\Delta 2.5\Delta 2.7\Delta \ldots \Delta 4.3\Delta 4.5\Delta 4.7\Delta \ldots \Delta \ldots \Delta 16\Delta 8\Delta 4\Delta 2\Delta 1$.

Šarkovskiĭ's Theorem [6,7] states that if $f \in C(I)$ and $n \in P(f)$ then $m \in P(f)$ when $n \Delta m$.

Let $f \in C(I)$ and $n>1$ be the minimum of $P(f)$ in the Δ-ordering. We say that a periodic orbit of f is minimal if their period is n.

Let $P=\{P_1,\ldots,P_n\}$ be a periodic orbit of $f \in C(I)$ of period $n=2^m q > 1$, where $q \geq 1$ is odd, $m \geq 0$ and $P_1<P_2<\ldots<P_n$. For $q>1$ and any integer $m \geq 0$, we define a simple periodic orbit inductively. Suppose $m=0$ and let $t=(q+1)/2$, then we say P is simple if either (a) or (b) holds:

(a) $f(P_{t-k})=P_{t+k+1}$ for $k=0,1,\ldots,t-2$

$f(P_{t+k})=P_{t-k}$ for $k=1,2,\ldots,t-1$ and

$f(P_1) = P_t$

(b) $f(P_{t-k})=P_{t+k}$ for $k=1,2,\ldots,t-1$

$f(P_{t+k})=P_{t-k-1}$ for $k=0,1,\ldots,t-2$ and

$f(P_n) = P_t$.

Now suppose $m \geq 1$. Then we say P is simple if the two subsets $\{P_1,\ldots,P_{n/2}\}$ and $\{P_{n/2+1},\ldots,P_n\}$ of P are simple periodic orbits of f^2. Then we have $f(\{P_1,\ldots,P_n\})=\{P_{n/2+1},\ldots,P_n\}$. Finally, for $q=1$ and $m \geq 1$, we also define a simple periodic orbit inductively. If $m=1$, then P is simple. Suppose $m>1$, then we say p is simple if the two subsets $\{P_1,\ldots,P_{n/2}\}$ and $\{P_{n/2+1},\ldots,P_n\}$ of P are simple periodic orbits of f^2.

It is easy to find that the number of possible different behavior of the simple periodic orbits ot period $n=2^m q$ where $m \geq 0$ and $q \geq 1$ odd is: 2^{2^m-m-1} if $q=1$ and $2^{2^{m+1}-m-1}$ if $q \geq 3$.