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THE MONOTONICITY OF THE ENTROPY FOR A FAMILY OF DEGREE ONE CIRCLE MAPS

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ABSTRACT. For the natural biparametric family of piecewise linear circle maps with two pieces we show that the entropy increases when any of the two parameters increases. We also describe the regions of the parameter space where the monotonicity is strict.

1. STATEMENT OF THE RESULTS

In this paper we study the monotonicity of the entropy for a biparametric family of degree one circle maps. The monotonicity of the entropy for particular families of maps of the interval has been considered by several authors for several families (see [MV, BMT, MT, DH]). We consider a problem similar to the one considered in [MV]. We deal with the biparametric family of piecewise linear circle maps with two pieces and we prove that the entropy increases when any of the two slopes increases. We also describe the regions of the parameter space where the monotonicity is strict.

In [AM] a kneading theory for a class of bimodal continuous circle maps of degree one (called class $\mathscr{A}$) was developed. The framework of the present study will be that kneading theory. Therefore, this paper has to be considered as a second part of [AM]. Hence, we assume the reader is familiar with the notation, definitions, proofs, and techniques developed in [AM] and we shall use them freely in this paper.

The family we are going to study can be defined as follows (see Figure 1). For $\lambda > 1$ and $\mu > 0$ we set

$$G_{\lambda,\mu}(x) = \begin{cases} \lambda x & \text{if } x \in [0, \frac{\mu+1}{\mu+\lambda}], \\ 1 + \mu(1-x) & \text{if } x \in [\frac{\mu+1}{\mu+\lambda}, 1], \\ E(x) + G_{\lambda,\mu}(D(x)) & \text{if } E(x) \neq 0, \end{cases}$$

(where $E(\cdot)$ denotes the integer part function and $D(\cdot)$ the decimal part function, i.e., $D(x) = x - E(x)$).

Clearly $G_{\lambda,\mu} \in \mathscr{A}$, $c_{G_{\lambda,\mu}} = \frac{\mu+1}{\mu+\lambda}$ and $G_{\lambda,\mu}(0) = 0$ for all $\lambda > 1$ and $\mu > 0$. Hence $\underline{I}_{G_{\lambda,\mu}}(0)$ remains constant when the parameters vary. On the other hand,

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