MONOTONICITY OF THE ENTROPY FOR A FAMILY OF CIRCLE MAPS*

LLUÍS ALSEDÀ[1] AND FRANCESC MAÑOSAS[2]

[1] Departament de Matemàtiques,
Facultat de Ciències,
Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Spain

[2] Departament de Matemàtica Aplicada II,
E.T.S d'Arquitectura del Vallès,
Universitat Politècnica de Catalunya
Ap. de Correus 508 (Terrassa)
Terrasa, Barcelona, Spain

Abstract. For a biparametric family of piecewise linear circle maps with two pieces we show that the entropy increases when any of the two slopes increases. We also describe the regions of the parameter space where the monotonicity is strict.

Monotonicity of the entropy for particular families of maps of the interval has been considered by several authors (see [MV], [BMT], [MT] and [DH]). We consider a similar problem to the one considered in [MV]. To define our family we introduce some notation.

As usual **Z**, **R** and **C** and will be the set of integers, real and complex numbers, respectively. We denote by $e : \mathbf{R} \longrightarrow S^1 = \{z \in \mathbf{C} : |z| = 1\}$ the natural projection $e(x) = exp(2\pi i x)$. A continuous map $F : \mathbf{R} \longrightarrow \mathbf{R}$ is called a *lifting* of a continuous map $f : S^1 \longrightarrow S^1$ if $e \circ F = f \circ e$. Note that if F is such a map then there is $k \in \mathbf{Z}$ such that $F(x+1) = F(x) + k$ for all $x \in \mathbf{R}$. This k is called the *degree* of F. Also we note that every lifting of f is of the form $F + m$ with $m \in \mathbf{Z}$.

We will denote by $\mathcal{L}$ the class of all liftings of continuous maps of the circle into itself of degree one; that is, the class of all continuous maps $F : \mathbf{R} \longrightarrow \mathbf{R}$ such that $F(x+1) = F(x) + 1$ for all $x \in \mathbf{R}$. It is easy to see that if $F \in \mathcal{L}$ then $F(x+k) = F(x) + k$ for all $x \in \mathbf{R}$ and $k \in \mathbf{Z}$ and that $F^n \in \mathcal{L}$ for all $n \geq 0$. Instead of considering continuous circle maps of degree one we shall work in the equivalent framework of class $\mathcal{L}$. For a map $F \in \mathcal{L}$, $h(F)$ denotes the topological entropy of the corresponding circle map (that is of the circle map f such that $e \circ F = f \circ e$). Also we shall denote by $L_F = [a_F, b_F]$ its rotation interval (see [NPT] and [I]).

*This is a summary of [AM2]