BIFURCATIONS FOR A CIRCLE MAP FAMILY ASSOCIATED WITH THE VAN DER POL EQUATION

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Levi had reduced the qualitative analysis of the van der Pol equation, essentially, to study a convenient family of circle maps. In this note we give information about the bifurcations of this family of circle maps.

1. Levi's results on the van der Pol equation

We study the following differential equation of the van der Pol type with periodic forcing

$$\varepsilon\ddot{x} + \phi(x)\dot{x} + \varepsilon x = bp(t) \qquad (1)$$

where ε is a small but fixed parameter, $p(t)$ is a periodic function of period T, $\phi(x)$ and $p(t)$ are in C^1-neighborhood of the functions $\phi_0(x) = \operatorname{sgn}(x^2-1)$ and $p_0(t) = \operatorname{sgn}\sin(2t/T)$, and b belongs to some finite interval $[b_1,b_2]$ (independent of ε). Levi describes the nonautonomous flow (1) by using the Poincaré map

$$D:\ (x,\dot{x})_{t=0} \longrightarrow (x,\dot{x})_{t=T}$$

In [5] one shows that the range $[b_1,b_2]$ of b-values consists of the alternating open subintervals A_k, B_k separated by the gaps g_k of small (with ε) total length, such that the qualitative behavior of the map D throughout each interval A_k, B_k is preserved, while g_k are the bifurcation intervals. For all $b\in[b_1,b_2]$, D has one totally unstable fixed point z_0; furthermore,

(A) for $b\in A_k$, has exactly one pair of periodic orbits of period $2n-1$ with $n = n(k) \approx 1/\varepsilon$ constant throughout each A_k. One of these orbits is a sink, another a saddle.

(B) for $b\in B_k$, the invariant set of D consists (besides z_0) of two sink-saddle periodic orbits of periods $2n+1, 2n-1$ correspondingly, and of an invariant hyperbolic Cantor set wich is given by using a subshift of finite type. Both cases: $b\in A_k$ and $b\in B_k$ correspond to D structurally stable.

(C) as b crosses the gap g_k a complicated sequence of bifurcations occurs.