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Twist periodic orbits and topological entropy for continuous maps of the circle of degree one which have a fixed point

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Abstract. Let f be a continuous map from the circle into itself of degree one, having a periodic orbit of rotation number $p/q \neq 0$. If $(p, q) = 1$ then we prove that f has a twist periodic orbit of period q and rotation number p/q (i.e. a periodic orbit which behaves as a rotation of the circle with angle $2\pi p/q$). Also, for this map we give the best lower bound of the topological entropy as a function of the rotation interval if one of the endpoints of the interval is an integer.

1. *Introduction and results*

Let S^1 be the circle. We denote by $C_1(S^1)$ the set of all continuous maps from S^1 to itself of degree one. For $x \in S^1$, we say that x is periodic if there exists a positive integer n such that $f^n(x) = x$. The period of x is the smallest integer satisfying this relation. Let $P(f)$ be the set of periods of f. If $x \in S^1$ is a periodic point of period n, then the orbit of x is the set $\{f^k(x): k = 1, 2, \ldots, n\}$. We refer to such an orbit as a periodic orbit of period n.

Let $f \in C_1(S^1)$, F its lifting to the covering space $\mathbb{R}$ and $e(X) = \exp(2\pi iX)$ the natural projection of $\mathbb{R} \to S^1$. We note that F is not defined uniquely; nevertheless, if F and F' are two liftings of f then $F = F' + m$ with $m \in \mathbb{Z}$. Since $\deg(f) = 1$ we have $F(X+1) = F(X) + 1$ for all $X \in \mathbb{R}$. If x is a periodic point of f of period n and $e(X) = x$, then $F^n(X) = X + k$ where $k \in \mathbb{Z}$. We shall call k/n the *rotation number* (or F-rotation number, if necessary) of x and we denote it by $\rho(x)$ or $\rho_F(x)$. We denote by $L(f)$ or $L_F(f)$ the set of all rotation numbers of f. The following statements are known (see [2] and [5]):

(1) $\rho(x)$ does not depend on the choice of X. Actually, it depends on the periodic orbit.