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Lower bounds of the topological entropy for continuous maps of the circle of degree one

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Abstract. We give the best lower bound of the topological entropy of a continuous map of the circle of degree one, as a function of the rotation interval. Also, we obtain as a corollary the theorem of Ito, which gives the best lower bound of the topological entropy depending on the set of periods.

1. Introduction and results

To understand the complicated dynamics of various systems it is sometimes very useful to reduce the problem to the study of iterates of relatively simple maps, like maps of one-dimensional spaces, mainly of the interval or the circle. For example many properties of the van der Pol equation can be reduced to those of a four-parameter family of circle maps of degree one (see Levi 1981). For a large class of diffeomorphisms a similar reduction is used by Newhouse *et al* (1983).

The main tool used to investigate circle maps of degree one is their rotation interval. This is a generalisation of the notion of rotation number, introduced by Poincaré for the study of circle homeomorphisms. Vaguely speaking, it is the set of average angular speeds of points moving around the circle under iteration of the map.

Topological entropy characterises the complexity of the maps. For a piecewise monotone map, it measures the exponential growth rate of the number of pieces of monotonicity of the iterates of the map (see Misiurewicz and Szlenk 1980). Roughly speaking, it also measures the exponential growth rate of the number of periodic orbits, as we increase their periods (Misiurewicz and Szlenk 1980).

The rotation interval also provides a lot of information about periodic orbits (see

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