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KNEADING THEORY OF LORENZ MAPS*

LLUIS ALSEDÀ[1,2] AND JAUME LLIBRE[1]

[1] *Departament de Matemàtiques, Facultat de Ciències*

[2] *Departament d'Economía i Història Econòmica, Facultat de Ciències Econòmiques*
Universitat Autonòma de Barcelona
08193 *Bellaterra, Barcelona, Spain*

1. Introduction

In the study of the geometrical model of the Lorenz attractor, a class of one-dimensional maps plays an important role. We refer to such maps as the Lorenz maps (see [GH], [Sp] and [T]) although they are different from the one-dimensional maps presented by Lorenz (see [L]). We describe the use of the kneading theory to study the dynamics of Lorenz maps.

Let $I = [-1, 1]$. We say that a map $f: I \to I$ is *Lorenz* if

(L1) f has a single discontinuity at 0, $\lim_{x \to 0^+} f(x) = -1$ and $\lim_{x \to 0^-} f(x) = 1$,

(L2) f is odd on $I \setminus \{0\}$ (i.e. $f(-x) = -f(x)$ for all $x \in I \setminus \{0\}$),

(L3) $f(-1) < 0$ and $f(0) = 1$,

(L4) f is once continuously differentiable on $I \setminus \{0\}$, and $f'(x) > 1$ for all $x \in I \setminus \{0\}$.

Note that every Lorenz map is strictly monotone on the intervals $[-1, 0)$ and $(0, 1]$. Many of our results would also be true for maps satisfying (L1), (L2), (L3) and

(L5) f is strictly increasing on $[-1, 0)$ and $(0, 1]$,

instead of (L4); but the ideas of some proofs seem more transparent for Lorenz maps. It is also easy to see that the particular choice of the interval $[-1, 1]$ and fixing the discontinuity at $x = 0$ involves no loss of generality.

Let $J = [-1, 0]$. We say that a map $g: J \to J$ is *piecewise expanding unimodal* if

(U1) g is continuous,

(U2) there exists $c \in (-1, 0)$ such that $g(c) = 0$,

(U3) $g(0) = -1$,

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* This is a summary of [AL].