# MINIMAL PERIODIC ORBITS AND TOPOLOGICAL ENTROPY OF ONE DIMENSIONAL MAPS

Memoria presentada per Lluís Alsedà i Soler per a aspirar al grau de Doctor en Ciències, Secció de Matemàtiques (per la Universitat Autònoma de Barcelona)

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CERTIFICA:

Que la present Memoria ha estat realitzada sota la seva direcció per Lluís Alsedà i Soler, i que constitueix la seva tesi per a aspirar al grau de Doctor en Ciències, Seccio Matemàtiques.

Bellaterra a 4 de Juny de 1984.

*A la Dolors,*
*que ha suportat l'elaboració*
*d'aquesta tesi amb resignació exemplar.*

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# INTRODUCTION

For continuous maps of the interval into itself, Sarkovskii's Theorem gives the notion of minimal periodic orbit. In Chapter I, we complete the characterization of the behavior of the minimal periodic orbits. Also in Chapter I, we show for the unimodal maps, that the min-max describes essentially the behavior of the minimal periodic orbits. In the Appendix to Chapter I we summarize the definitions and results from the book of Collet and Eckmann that we use in the study of the minimal periodic orbits of a unimodal map.

The main result of Chapter II, is to complete Misiurewicz's characterization of the set of periods of a continuous map f of the circle with degree one ( which depends on the rotation interval of f). As a corollary, we obtain a kind of perturbation theorem for maps of the circle of degree one, and a new algorithm to compute the set of periods when the rotation interval is known. Also, for maps of degree one which have a fixed point, we describe the relationship between the characterizations of the set of periods of Misiurewicz and Block. These ideas will be used in Chapter III.

In Chapter III we extend the notion of minimal periodic orbit, given for continuous maps of the interval, to continuous maps of the circle having a fixed point. The main result of this chapter is to characterize the shape of these minimal periodic orbits. As a consequence, we improve the known lower bound of the topological entropy,for a special class of maps of the circle of degree 0 or -1.

The Appendix to Chapters III and IV summarizes the techniques given in the so called paper of Block, Guckenheimer, Misiurewicz and Young, to compute lower bounds of the topological entropy of maps of the interval and of the circle. We use these techniques in Chapters III and IV.

Let f be a continuous map from the circle into itself of degree one, having a fixed point with rotation number 0 and a periodic orbit with rotation number $p/q \neq 0$. If $(p,q) = 1$, in Chapter IV, we prove that f has a twist orbit with period q and rotation number p/q ( i.e., a periodic orbit which behaves as a rotation of the circle with angle $2\pi p/q$). Also, for such a map, we give the best lower bound of the topological entropy,as a function of the rotation interval,if one of the endpoints of the interval is an integer. In this way we obtain better lower bounds of the topological entropy of continuous maps of the circle of degree one, than those given in [BGMY]and [BCN].

In Chapter V, under the assumption that the conjecture is true, we characterize the lower bounds of the topological entropy of a continuous map of the circle of degree one, depending on its rotation interval. That is, we extend Theorem B of Chapter IV to the general case.

Ito gave a formula to obtain lower bounds of the topological entropy, for continuous maps of the circle of degree one without fixed points, depending on the set of periods of the map ( see

[Il] ). In the Appendix to Chapter V we show that, if Conjecture 1 is true, Ito's formula is a particular case of the formula given in Chapter V.

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