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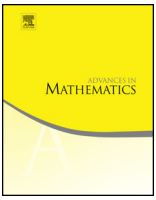

Quadratic planar differential systems with algebraic limit cycles via quadratic plane Cremona maps

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ABSTRACT

In this paper we study the action of planar birational transformations, also known as plane Cremona maps, on quadratic planar differential systems. We provide geometrical characterizations of when a quadratic system is transformed into a new quadratic system after applying a quadratic plane Cremona map. These conditions are expressed in terms of local properties of the plane Cremona map at the singular points of the system. As a consequence, we provide a new family of quadratic systems having an algebraic limit cycle of degree 5. Moreover we classify the known families of quadratic differential systems having an algebraic limit cycle by the action of quadratic plane Cremona maps. We also provide the phase portraits on the Poincaré disk of all these families.

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