

The Period Function of Hamiltonian Systems with Separable Variables

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Abstract

In this paper we study the period function of those planar Hamiltonian differential systems for which the Hamiltonian function H(x, y) has separable variables, i.e., it can be written as $H(x, y) = F_1(x) + F_2(y)$. More concretely we are concerned with the search of sufficient conditions implying the monotonicity of the period function, i.e., the absence of critical periodic orbits. We are also interested in the uniqueness problem and in this respect we seek conditions implying that there exists at most one critical periodic orbit. We obtain in a unified way several sufficient conditions that already appear in the literature, together with some other results that to the best of our knowledge are new. Finally we also investigate the limit of the period function as the periodic orbits tend to the boundary of the period annulus of the center.

Keywords Hamiltonian differential system \cdot Center \cdot Period function \cdot Critical periodic orbit

Mathematics Subject Classification 34C25 · 37J35 · 70H05 · 70K42 · 70K70

1 Introduction and Definitions

The present paper deals with the class of planar Hamiltonian differential systems

$$\begin{cases} \dot{x} = -H_y(x, y), \\ \dot{y} = H_x(x, y), \end{cases}$$

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